OPTIMAL LIMIT STATE DESIGN OF REINFORCED CONCRETE SHAFT SUPPORTED CONICAL WATER TANKS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

DOCTOR OF PHILOSOPHY

By

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to the

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Dedicated

to

The Divine Mother

To Be on the Path

What I call "to be on the path" is to be in a state of Consciousness wherein only union with the Divine has any value, to live this union is the one thing worthwhile, the only object of aspiration; everything else has lost all value and is not worth the seeking; since there is no longer an object of desire, there is no question of renunciation.

As long as the union with the Divine is not *the* thing for which one lives, one is not yet on the path.

THE MOTHER

CERTIFICATE

This is to certify that the thesis "Optimal Limit State Design of Reinforced Concrete Shaft Supported Conical Water Tanks" submitted by Shri M.L. Kalwar in partial fulfilment of the requirements for the degree of Doctor of Philosophy of the Indian Institute of Technology, Kanpur, is a record of bonafide research work carried out by him under our supervision and guidance. The work embodied in this thesis has not been submitted elsewhere for a degree.

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- M.L. Kalwar

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A	area of cross-section
A_s	area of tension reinforcement
a_{cr}	distance from point under consideration to the surface of the nearest longitudinal bar
B	extensional stiffness of shell per unit width
b_t	width of the member at the level of tension reinforcement
C_c	unit cost of finished concrete
C_f	unit cost of formwork including labour
C_s	unit cost of steel including the costs of cutting, bending and laying
c_{min}	minimum cover to reinforcement
c_1, c_2 etc.	constants of integration
D	bending stiffness of shell per unit width
d	effective depth of the member
d_n	depth of neutral axis
E	modulus of elasticity of concrete
F	objective function
F', F''	objective function of parts of the structure
F_i	constant factors for shells
$F_i(n)$	variable factors for shells
F_o	seismic zone factor
f	height of water level from a reference point
f_{ij}	coefficients of flexibility matrix
f_y	characteristic strength of reinforcement
g	constraint; acceleration due to gravity
H	corrective horizontal force per unit length acting at the junction of shells

$[H]$	a positive definite symmetric matrix
h	depth of the member
I	second moment of area; a coefficient depending upon the importance of the structure
i	iteration number
j	constraint number
K	a constant
K_1, K_2 etc.	constants
k	cycle number
L	length of supporting cylindrical shaft
L_{cy}	length of inner cylindrical shaft
L'	a reference length for cylindrical shell
$\bar{L}$	length of conical tank wall
l	height of partition wall
l_1, l_2	reference lengths for conical and cylindrical shell
M	corrective moment per unit length acting at junction of shells
M_x	longitudinal bending moment in the shell
M_ϕ	meridional bending moment in the shell
M_θ	circumferential bending moment in the shell
M_o	moment of resistance of shell per unit width to resist positive moment
M'_o	moment of resistance of shell per unit width to resist negative moment
$[M]$	an updating matrix
m	number of constraints
N_c	hoop compression capacity per unit width
N_t	hoop tension capacity per unit width
N_x	longitudinal membrane force in the shell

N_ϕ	meridional membrane force in the shell
N_θ	hoop membrane force in the shell
$[\underline{N}]$	an updating matrix
n	number of design variables; a normalised reference variable for conical or spherical shell
$\bar{n}$	a normalised reference variable for cylindrical shell
P	equally distributed vertical load per unit length along the edge of the shell
p_b	basic wind pressure
p_{bd}	percentage of reinforcement for bottom dome
p_{rc1}, p_{rc2}, p_{rc3}	percentages of hoop reinforcement in the conical tank
p_{rc4}	percentage of longitudinal reinforcement for conical tank for a short length
p_{rcy}	percentage of longitudinal reinforcement for inner cylindrical shaft for a short length
p_{rmin}	minimum percentage of reinforcement
p_{rp1}, p_{rp2}, p_{rp3}	percentages of hoop reinforcement for cylindrical partition
p_{rp4}	percentage of vertical reinforcement for cylindrical partition for a short length
p_{rs1}, p_{rs2}, p_{rs3}	percentages of vertical reinforcement for cylindrical supporting shaft
p_x	water pressure at section X
p_o	collapse pressure
Q_x	longitudinal shear force in the shell
Q_ϕ	meridional shear force in the shell
$\bar{Q}$	gradient difference vector
q	self weight of the shell per unit surface area
R	radius of the shell

R_1	radius of the conical tank at larger end
R_2	radius of the supporting shaft
R_{bd}	radius of the bottom dome
R_{cy}	radius of the inner cylindrical shaft
R_d	radius of the top dome
r	penalty parameter
$\bar{S}$	search direction
T	natural period of vibration
t	thickness of the shell
t_{bd}	thickness of the bottom dome
t_c	thickness of the conical tank
t_{cy}	thickness of the inner cylindrical shaft
t_d	thickness of the top dome
t_{min}	minimum thickness of the shell
t_s	thickness of the supporting shaft
W	weight of the water tank under full or empty condition
W_p	design wind pressure at any level
w	radial displacement of the cylindrical partition
w_{cr}	estimated width of surface flexural cracks
w_{cra}	allowable width of crack
$\bar{X}$	vector of unknown corrective edge forces; vector of design variables
x	reference length
x_1, x_2	reference lengths for conical and cylindrical tanks
x_m	reference length for conical tank
y	distance from neutral axis to the level of point under consideration

α	inclination of conical tank wall with horizontal; step length
α'	reference angle for the spherical shell
α_h	horizontal seismic coefficient
α_o	basic seismic coefficient
α^*	minimizing step length
β	rotation at any point of the shell; a coefficient depending upon the soil-foundation system
γ_L	partial safety factor for loads
θ	a reference angle for circumferential direction
$\emptyset$	a reference angle for meridional direction; modified objective function
$\emptyset_1, \emptyset_2$	reference angles for spherical shell
χ_x	generalised strain associated with M_x
λ_θ	generalised strain associated with N_θ
Δ	the static horizontal deflection at the top of tank under a static horizontal force equal to a weight W acting at centre of gravity of tank
$\bar{\Delta}_p$	deformation vector due to primary loadings
ρ	unit weight of water
ρ_c	unit weight of concrete
ρ_s	unit weight of steel
μ	Poisson's ratio
σ	uniform stress
ε_s	strain in steel at service loads
$\nabla\emptyset$	gradient of the modified objective function
δ	horizontal displacement at any point of the shell.

SYNOPSIS

OPTIMAL LIMIT STATE DESIGN OF REINFORCED CONCRETE SHAFT SUPPORTED CONICAL WATER TANKS

(A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy by M.L. Kalwar to the Department of Civil Engineering, Indian Institute of Technology, Kanpur, India)

Water towers are the land marks of any modern day habitat. Unlike in the past, the present day society expects, rather demands the modern structures to be shaped in such a way that whole environment is enhanced by their very presence. In a developing country like India wherein the resources are limited, a structural designer has to make a compromise between the aesthetic and economic aspects. A conscientious designer has to proportion the structure in such a fashion that structural efficiency and economy are achieved simultaneously. Hence the role of a present day structural designer is all the more exacting as he has to design a structure which satisfies the requirements of safety, functional aspects, economy and aesthetics. The choice of a structural material depends upon its initial and the maintenance costs. Hence reinforced concrete water towers are preferred over others. A water tower comprises of a tank or a reservoir, a supporting structure and the foundation. The present study addresses itself to the superstructure only viz. the tank and its supporting structure.

The water towers considered in the present study are multishell structures consisting of a conical tank supported

on a cylindrical shaft with a spherical top. These towers are classified as Type I, II and III depending upon the capacity and the internal geometric arrangement of the elements in the reservoir portion. A multishell structure may be composed of simple shell and circular ring elements. The elements considered herein are axisymmetric in nature.

Traditionally the reinforced concrete structures are designed either by working stress method based on elastic theory or by ultimate load theory. Both of these methods are found to be inadequate to provide a rational design. It is also realised that the loads and the material properties are after all not deterministic. The foregoing considerations set the stage for the development of a much needed, more rational and sound design philosophy. The limit state design concept is a consequence of this. With the evolution of this new concept of design both the elastic and plastic methods of analysis together are brought into limelight. The codes of practice around the Globe are being revised to incorporate the limit state theory, and Indian Standards Institution is no exception.

In the present study, the optimal limit state design of reinforced concrete, shaft supported, conical water tanks is investigated using the limit state method for Indian environmental conditions. In India many water supply schemes are coming up and obviously a large number of water towers will have to be constructed to meet the requirement. Hence optimal design of such structures will be very meaningful.

The response of a structure to service loads is predicted by elastic analysis and the resistance to ultimate

loads obtained through limit or plastic analysis. The design forces in the reservoir portion are found to be governed by axisymmetric loading combination of dead load and water loading under full tank condition. For this loading combination elastic analysis of the tower is carried out by using force method of analysis, which is specially suited to axisymmetrically loaded shells of revolution. However lateral forces due to wind and seismic effect under full or empty tank condition are found to be critical in case of supporting shaft. Hence for simplification the shaft is idealised as a cantilever for purposes of analysis. For computing the seismic forces, response spectrum method has been used.

For the proper application of the limit state theory, the resistance of the structure to ultimate loads is to be ascertained. Therefore, various collapse mechanisms of reinforced concrete conical shell subjected to a hydrostatic pressure, adopting the upper bound approach, are investigated. All the possible mechanisms are considered to arrive at an exact value of collapse pressure. In Type III water towers, in addition to conical reservoir, the collapse mechanisms of cylindrical partition are also investigated to arrive at the collapse water pressure. The resistance of the supporting shaft to ultimate loads is determined by carrying out its limit analysis.

All the strength and serviciability constraints of the relevant limit states are incorporated in the nonlinear programming model of the formulated optimum design problem.

The shaft diameter, ratio of top diameter to bottom diameter of the conical reservoir, thicknesses of various elements and the percentages of reinforcement in these elements, with practicable logical distribution of reinforcement, are taken as design variables. The overall cost of the superstructure of the tower, which includes the costs of material, formwork and labour, is chosen as the objective function for the optimal design. The optimization study of these towers yields not only the optimal value of the design variables but also the optimal configuration of the tower. Such an investigation has been carried out for the first time. Furthermore, the sensitivity of the optimal design with respect to the escalation of unit costs of concrete, steel or formwork has been studied in the present work.

Parametric studies have been conducted for the most commonly used reservoir capacities and staging heights. In order to obtain designs for various wind zones of India, the basic wind pressure is also treated as a parameter in the study of optimal design of these water towers. The suitability of these designs, obtained for various wind zones, are established for the different seismic zones.

In order to save the computational time the optimization is carried out using an indirect approach to arrive at the optimal configuration. This approach has not only resulted in considerable saving of computational time but also gives designs with optimal design variables and the optimal ratio of top diameter to the bottom diameter of the conical

reservoir for different shaft diameters, in addition to the one which corresponds to the optimal configuration.

For the benefit and easy reference of the field engineers, designs are presented in tabular form from which the optimal values of design variables may be picked up for a particular tank capacity, staging height, wind and seismic zones. Thus the designer will be able to arrive at an optimal design which conforms to the relevant codes of practice and without recourse to a powerful computing device. The computations have been carried out with the standardisation of form in view. Thus, the optimal solutions presented are the realistic solutions embodying the field considerations.

Although the present study is confined to Indian environment, the results can be judiciously utilized for other environments as well. In essence the thesis contains optimal limit state designs of reinforced concrete shaft supported conical water tanks, which are useful to a professional engineer.

CHAPTER 1

INTRODUCTION

1.1 General

The basic aim of a structural engineer is to design such structural systems which satisfy safety, functional, aesthetic, and economic requirements in the most efficient manner. In order to achieve the best results, one has to adopt such a methodology, for analysis as well as design, which is capable of predicting the realistic behaviour of a structure when subjected to various design loads and gives satisfactory design which remains safe and serviceable throughout its design life. However, if a structural theory is required to take into account every possible variable involved, it will become too complicated for practical use. So the general practice is to make simplifying assumptions that yield consistent and sufficiently accurate results. Experience, experiments and basic understanding often are required to determine whether a given theory or method is applicable to a particular situation. Neither 'working stress design method' nor 'ultimate load design theory' has been found to be adequate for design of reinforced concrete structures. Moreover, to take into account the variations that are likely to occur in the loads to which the structures are subjected and the variations in the strength of material of which they are comprised, a probabilistic approach to safety has to be adopted.

No doubt, 'limit state design philosophy' eliminates the limitations of working stress and ultimate load design methods and is capable of giving satisfactory designs. But, in order to ensure economy and efficiency in such designs, optimization techniques have to be incorporated in the design process.

Recent advances in computer-aided analysis and design of concrete structures have made it possible to analyse and design complex concrete structures under the expected static and dynamic loadings. The computer is an important engineering tool allowing considerable automation in the standardised design of conventional concrete structures. Optimization becomes more meaningful when used for such standardised designs.

The availability of a very good computer facility at the Indian Institute of Technology, Kanpur and the foregoing considerations have motivated the present study dealing with optimal limit state design of reinforced concrete water towers.

1.2 The Present Study

In a developing country like India one of the most important basic needs is to provide adequate protected water supply systems to industrial and residential complexes in cities, towns and villages. Water towers are among the major constructions of most water supply systems. Many water supply schemes are coming up throughout the country and obviously a large number of water towers will have to be constructed to fulfil the need. In the recent past many reinforced concrete

water towers, particularly with cylindrical shell type support, have collapsed in India. This has resulted in some of the organisations losing faith in the construction of shaft overhead tanks. On investigation of some of these failures, it was observed that a number of reasons are responsible for this sorry state of affairs. The principal reasons identified are,

(i) lack of understanding of the behaviour of the structural system;

(ii) the ignorance of the tolerance limits on the geometric parameters; and

(iii) the use of substandard formwork in the construction stage.

In view of these facts a deeper insight into the structural behaviour and design of such structures is sought for. Moreover, not much of work has been done on optimization of water towers using either elastic or plastic models.

In the light of the foregoing discussions, the obvious reasons for undertaking the present study may be summarized as:

(i) To analyse and design reinforced concrete water towers using modern numerical tools which allow a useful insight into the structural behaviour and design aspects of such structures under the expected static and dynamic loadings. As the number of such structures to be constructed is very large, any small saving that can be made in the design will get

added up. Hence optimization technique is incorporated in the design process.

(ii) To provide, for the benefit of the field engineers, optimal standardised designs of reinforced concrete water towers with various capacities and staging heights for different zones of India based on wind and seismic forces. This will make the designers free from substantial tedious work. Besides, standard forms may be designed for repeated use.

(iii) To instil interest among the academic community that such optimal studies usually provide, specially in areas where not much work had been done.

1.2.1 Choice of the shape for the water towers

Water towers are among those structures which come under the category of prominent features of a particular locality. Besides economy the aesthetic value of such structures is also of considerable importance and hence a proper consideration should be given to this aspect while choosing a particular shape for such structures. Keeping this fact in view, a conical reservoir with a cylindrical support has been chosen, in the present study, as the design configuration for the water towers. Such a configuration, undoubtedly, is aesthetically pleasing and has better architectural expressiveness than Intze tank or cylindrical tank supported either on columns or shaft. Moreover, this configuration does not pose any extra difficulty in construction.

1.2.2 Capacities, staging heights and the types of tanks considered

The scope of the present study has been kept limited to the designs of reservoirs of capacities ranging from 100 kl to 1000 kl on stagings of 15 m, 20 m and 25 m heights as these will be among the most commonly used capacities and heights. Reservoir capacities considered in the present study are 100, 200, 300, 500, 750 and 1000 kl. Depending upon the capacity and other requirements the internal geometric details of the reservoir are fixed without any change in the overall configuration. In the present study, based on the internal details of reservoirs, these water towers have been classified into three categories as Type I, Type II and Type III. The Figure 1.1 shows the details of these towers.

Type I --

This category shown in Figure 1.1(a) is intended for the reservoirs of capacities 100 to 300 kl. Reservoirs upto 300 kl capacity require supporting shaft of small diameter. Hence, in this case, diameter of internal shaft is kept same. This will result in easy constructional details without losing much of space of the reservoir.

Type II --

For reservoirs of capacities 500 to 1000 kl, towers of the type shown in Figure 1.1(b) are usually preferred. As the diameter of supporting shaft required for these capacities will be more, reservoirs are provided with spherical bottom and internal shaft having smaller diameter as compared

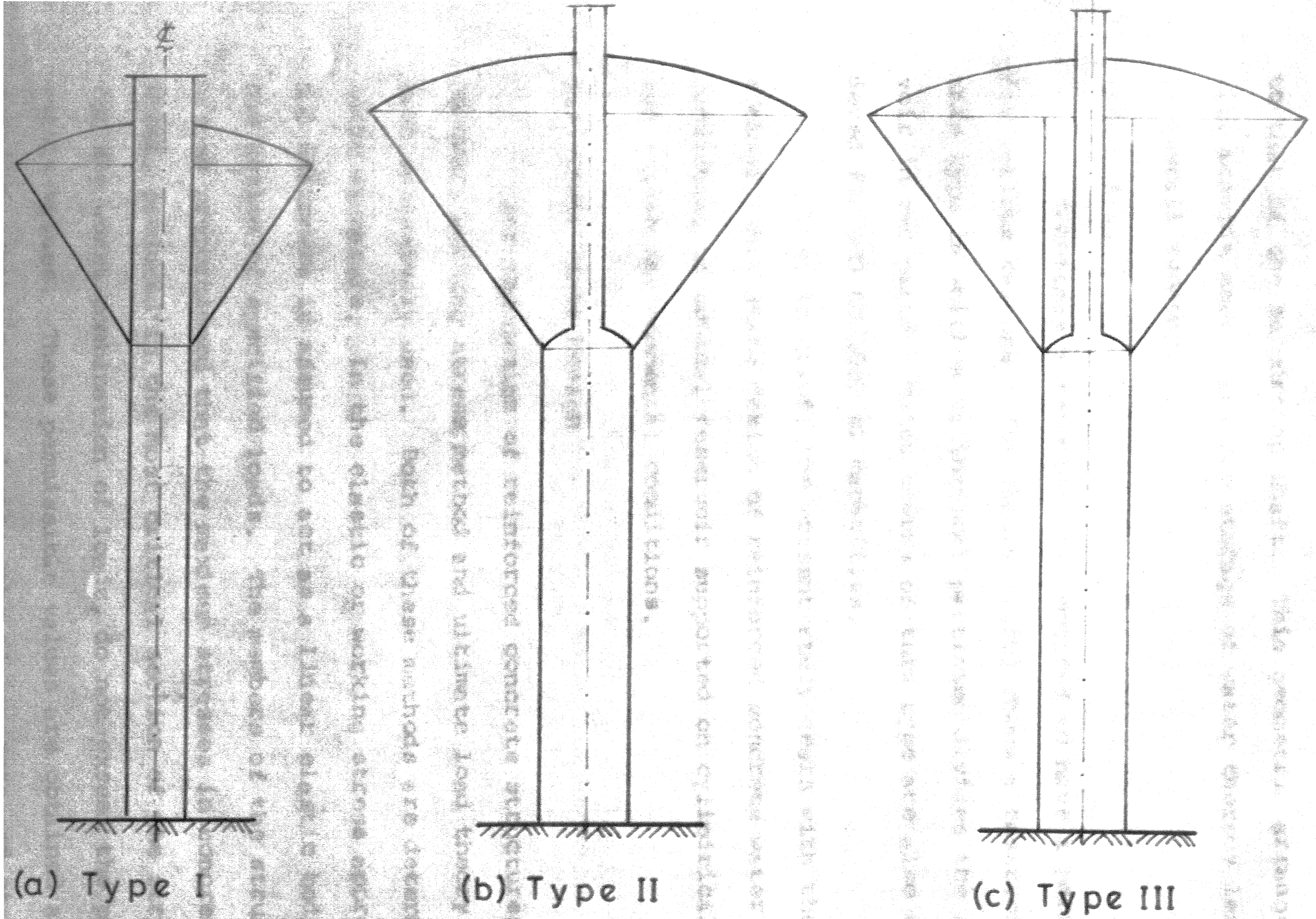


Fig.1.1 Types of water towers considered

to that of the supporting shaft. This geometric arrangement will provide more space for storage of water thereby leading to overall economy.

Type III --

Sometimes a reservoir is required to have a partition for various reasons. The Figure 1.1(c) shows a reservoir of this type in which a cylindrical partition divides the reservoir in two parts. Water towers of this type are also considered for 500 to 1000 kl capacities.

Thus, in brief, the present study deals with the optimal limit state design of reinforced concrete water towers, consisting of conical reservoir supported on cylindrical shaft, for Indian environmental conditions.

1.3 Limit State Design

For the design of reinforced concrete structures, in general, working stress method and ultimate load theory have been extensively used. Both of these methods are deterministic in nature. In the elastic or working stress approach, the structure is assumed to act as a linear elastic body under the action of specified loads. The members of the structures are so proportioned that the maximum stresses in concrete and steel, produced in the most critical section of the structure, for the worst combination of loads, do not exceed the permissible values. These permissible values are obtained as a fraction of the specified strengths of concrete and steel. Many structures have been successfully designed by this method. The objections to this method are not that it is unsafe, but

that it is physically unreal and leads unnecessarily to conservative designs with varying degrees of safety against collapse. The maximum stress may occur at only one point in the whole structure which means that safety is being assessed by looking at a very local effect. It is also recognised that reinforced concrete deviates from elastic behaviour appreciably even at lower loads and radically at higher loads. Hence elastic analysis will be much less valid in case of reinforced concrete structures.

Foregoing limitations of the elastic analysis led, in the post-war years, to the development of the earlier method of ultimate load theory which had been abandoned with the advent of the theory of elasticity. According to the ultimate load theory, also termed as plastic or limit analysis, failure does not occur because of the stress reaching the limiting value at one point. On the other hand, it takes into account the inelastic behaviour of the material and redistribution of stresses. Experimental results have confirmed that the mode of collapse and load producing it are remarkably well predicted by this method. The margin of safety against collapse, called the load factor, can be chosen as per the requirement. Another contribution of this method is that it has diverted the attention from the microscopic stress concept to the macroscopic overall behaviour of the structure. Although this is an excellent model for finding the strength of a structure, it has a serious limitation that it cannot predict the response of the system to service loads. It is observed that some of the most

common causes of damage in reinforced concrete structures are excessive deflection, cracking etc. which make the structure unserviceable. Hence safety against these damages, demand due considerations in the design.

Thus, it is recognised that elastic theory is better suited to predict the response of a structure under service loads only and the ultimate strength can be determined only through limit analysis. As pointed out earlier, both these methods are deterministic in concept. The randomness involved in the basic design parameters has rendered the deterministic approach obsolete. The foregoing considerations set the stage for the development of a much needed, more rational and sound design philosophy. The limit state theory is a consequence of this and has been adopted for the design of structures considered in the present study.

1.3.1 Definition of limit states

A structure, or a part of a structure, is rendered unfit for use when it reaches a particular state, called a 'limit state' in which it ceases to fulfil the function, or to satisfy the condition, for which it was designed (TC98, 1973). Limit states can also be defined as: A condition where a structure or component ceases to fulfil its intended function in some way, or becomes unfit for its intended purpose.

Depending upon the safety, serviceability and other requirements, limit states may be grouped into three major categories.

(i) Limit states of strength:

Those corresponding to maximum load carrying capacity and may be governed by the rupture of critical sections, or excessive plastic deformation leading to collapse, or instability etc.

(ii) Limit states of serviceability:

Those corresponding to excessive deformations, wide cracking, excessive vibration etc. making the structure unserviceable.

(iii) Other limit states:

Those corresponding to fatigue, lightning, fire, explosion etc.

1.3.2 Safety formats for limit state design

Various national and international committees, working with the aim to provide a rational and sound design philosophy, have recommended a semi-probabilistic limit state method. Thus, probabilistic concepts, for the first time, are explicitly embedded in the design. It is assumed that the statistical distribution of loads and material strengths are available. The 'characteristic values' for both loads and material strengths are then selected with acceptable probabilities against higher load or lower strength respectively. The practice of using a single global factor of safety has been replaced by probability-based partial safety factors, one for loads and the other for material strengths. The 'design values' are derived from the characteristic values through

the use of partial safety factors. The partial safety factors for loads vary with the degree of seriousness of the particular limit state being reached, probability of two or more loads occurring together, and the reliability of the structural theory being used. Whereas, the partial safety factors for material strengths take into account the difference between the strengths of controlled specimens and the material in the prototype structure. With these modified loads and material strengths, safety of the structure is investigated for the relevant limit states.

1.4 Optimization

1.4.1 Structural optimization

Any design philosophy demands that the components of the structure be so designed that performance requirements are satisfied. Such requirements are termed as behaviour constraints in optimization literature. Construction or manufacturing processes may also impose certain restrictions on the design which are termed as side constraints. The usual method of indirect design, which does not make use of any optimization technique, is a trial and error process. An initial design is intuitively selected and the analysis is carried out to check its safety. If any constraint is violated, or margin of safety is too high the design variables are suitably altered and the analysis is carried out again. This procedure is repeated till a satisfactory feasible design is obtained. Among the many possible feasible designs

of a structure, some designs are better than others and it is very difficult to choose the best one. On the other hand, the optimum seeking methods, also known as optimization techniques, start with an initial design and proceed systematically to find the best design based on certain criterion, such as minimum weight/cost or maximum stiffness etc. Moreover, the optimum design approach provides a rational basis for automatic design methods using a digital computer.

1.4.2 Optimization techniques

Basically there are two approaches to optimization, analytical and numerical. Usually analytical methods like variational calculus, Langrange multipliers etc. are employed to solve relatively simple optimization problems. With the advent of digital computers, numerical approach of optimization has become popular. These numerical approaches are available in the literature of mathematical programming and are capable of dealing with a variety of problems in the field of optimization. Linear, nonlinear, geometric, dynamic, quadratic and integer programming techniques fall under the category of mathematical programming.

Linear programming deals with a particular class of optimization problems in which objective function as well as constraints are linear functions of design variables. Most of the structural engineering design problems do not come in this category. Both objective function as well as constraints are nonlinear in nature in most cases of structural design problems. Hence the most suited and widely used technique,

for such problems, is the nonlinear programming one, which has been chosen for the current study as well.

The nonlinear programming problems can broadly be classified as

(i) unconstrained, or

(ii) constrained.

1.4.3 Unconstrained nonlinear programming

As the name implies, unconstrained nonlinear programming deals with the optimization of an objective function without any constraint. This approach can be applied to constrained optimization problems also, after transforming the original problem to an equivalent unconstrained problem. Thus, unconstrained nonlinear programming has become one of the most important mathematical tools for solving both unconstrained and constrained optimization problems.

Optimization is concerned with minimization or maximization of an objective function. Since the maximum of a function corresponds to the minimum of the negative of the same function, optimization can be stated as minimization of a function without any loss of generality. Thus, the unconstrained optimization problem can be stated as finding the vector $\overline{X}$ of the design variables which minimizes the objective function $F(\overline{X})$ where there are no restrictions on the choice of $\overline{X}$. All the unconstrained minimization methods are iterative in nature and hence they start from an initial trial solution $\overline{X}_1$ and proceed towards the minimum point in a sequential manner. Thus, if $\overline{X}_i$ is the approximation to the

minimum in the (i-1)th iteration, the improved approximation in the ith iteration is found from the relation

$$\bar{X}_{i+1} = \bar{X}_i + \alpha_i^* \bar{S}_i \tag{1.1}$$

where $\bar{X}_{i+1}$ = vector of design variables at the new point,

$\bar{X}_i$ = vector of design variables at the previous point,

$\bar{S}_i$ = a suitable direction of move along which the value of objective function decreases, and

α_i^* = a step length which minimizes $F(\bar{X})$ in the direction $\bar{S}_i$.

All the constrained minimization methods differ from one another only in their approach to find $\bar{S}_i$ and α_i^*. These methods can be classified into two broad categories as direct search or descent methods.

The direct search methods require only objective function evaluations and do not use the partial derivatives of the function in finding the minimum and hence are often called the nongradient methods. Random search method, univariate method, pattern search methods, simplex method are among the nongradient methods. These methods are, in general, less efficient than the descent methods. The descent methods are also known as gradient methods because they make use of derivatives of the objective function in addition to objective function evaluation in finding the minimum. Steepest descent method, Fletcher-Reeves method,

Newton's method, Davidon-Fletcher-Powell method are among the gradient methods.

The various methods used to find the minimizing step length α_i^* can be classified as elimination methods and interpolation methods. Fibonacci method and golden section method among the elimination methods, and quadratic interpolation and cubic interpolation method among the interpolation methods are more commonly used depending upon the mathematical nature of the optimization problem at hand.

1.4.4 Constrained nonlinear programming

Most of the engineering designs come under the category of constrained optimization problems. A constrained optimization problem can be stated in the standard form as:

$$\left.\begin{array}{l} \text{Find vector } \overline{X} \text{ of the design variables which minimizes} \\ \quad \text{the objective function } F(\overline{X}) \text{ subject to m constraints} \\ \quad g_j(\overline{X}) \leq 0 , \qquad j = 1, 2, \dots m \end{array}\right] \tag{1.2}$$

There are many techniques available for the solution of a constrained nonlinear programming problem. These methods can be classified into two broad categories, namely, the direct or indirect method. The methods of feasible directions and constraint approximation methods are among the direct methods. Indirect methods involve the transformation of constrained problem into an equivalent unconstrained minimization problem. The transformed problem is then handled by any of the methods of unconstrained minimization as a

sequence of unconstrained minimization problems. This is achieved either by transformation of variables or by using penalty function methods.

1.4.5 Choice of optimization technique

The choice for a particular method of optimization, in nonlinear programming, depends on the size and the nature of the problem. The efficiency and the stability of these methods are also very much dependent on the problem.

For the medium-size problem (10 to 50 variables) Davidon-Fletcher-Powell method, also known as variable metric method, has been found to be stable and efficient. As such, this method has been chosen for the current study to find out the search direction $\bar{S}$. Since this method makes use of gradient of the objective function, it has been coupled with the cubic interpolation method, which is usually the best of 1-dimensional minimization methods, to find the appropriate step length α^*. The interior penalty function has been used to transform the problem into an equivalent unconstrained optimization problem.

In interior penalty function method, a ϕ function is constructed by augmenting a penalty term to the objective function $F(\bar{X})$ as given by Eq. (1.3)

By the repeated unconstrained minimization of the ϕ-function for a sequence of decreasing values of the penalty parameter r_k ($k = 1, 2, 3 \ldots$), the solution converges to that of the original problem stated in Eq. (1.2). Hence, penalty function methods are also known as sequential unconstrained minimization techniques (SUMT). The penalty term is chosen in such a way that its value will be small at points away from the constraint boundaries and will tend to infinity as the constraint boundaries are approached. Thus, once the unconstrained minimization of ϕ-function is started with any feasible solution, the subsequent solutions generated will always lie within the feasible domain, since the constraint boundaries act as barriers during the minimization process. The reason for not carrying out the minimization of the ϕ-function in only one stage using a small value of penalty parameter r, so that the minimum of $\phi(\overline{X}, r)$ tends to the minimum of $F(\overline{X})$, is that ϕ will be a very complex function away from the optimum and the algorithm will not get initiated if r is small. However, near the optimum point, the ϕ-function is well behaved and permits computations to be carried out with a small value of r. Therefore, minimization is carried out as a sequence of unconstrained minimizations with decreasing values of r and the final design variable vector of the previous cycle is taken as the initial design variable vector for the next cycle.

1.5 Need for Optimal Design Tables

Optimal designs of structures with limited parametric studies result in not only economical and efficient designs but also in better understanding of the response of the structure and in identifying the factors which largely influence the cost or the efficiency of the structure. Thus, the incorporation of optimization concepts results in both economy and improved techniques.

In the field of structural design, optimization has essentially remained a pursuit for research (Goble and Moses, 1975). No doubt, economy and efficiency can only be ensured through optimization techniques. But, an average structural engineer, particularly in India, is neither familier with optimization methods nor can afford to pay for the necessary computer software and time to carry out optimization studies.

Therefore, for the structures considered in this study, parametric studies have been made and results are presented in the tabular form. In order to enable the design engineer to directly pick up the optimal design, optimization studies have been carried out within the framework of codes of practice. As far as possible, the specifications of Indian standard codes have been followed for carrying out those designs. However, in certain cases of discrepancies and recent developments which are not yet incorporated in Indian codes, BS 5337 and CP 110 are followed.

1.6 Organisation of the Thesis

The thesis is divided into seven chapters of which the first one gives the brief introduction of various analysis and design philosophies, optimization techniques besides identifying the scope of the present study.

In Chapter 2, a selective review of literature related to scope of the thesis is made. Based on literature review a need for further investigation on certain aspects has been identified.

Optimal limit state designs of Type I, Type II, and Type III water towers are dealt with in Chapters 3, 4 and 5 respectively. Chapter 3 constitutes the main body of the thesis which deals in detail the force method of elastic analysis for multishell structures limit analysis of the conical reservoir, and all other design aspects. The general layout of each of these chapters has been, a brief discussion of the design considerations followed by the methods of analysis, formulation of the optimization problem, presentation of optimal designs in the tabular form, discussion of the results of parametric study and an example to illustrate the method of using the design tables.

Chapter 6 deals with computational aspects in which the details of the optimization technique used in this study are discussed. Certain important observations made during this study, related to computation, are highlighted.

Finally, summary and the conclusions resulting from the study along with suggestions for future work are presented in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

The present study encompasses many fields such as the response of reinforced concrete structures to service loads, limit analysis of structures, limit state philosophy and optimization. Literature review on some of these fields has been well documented and available even in some textbooks (Save and Massonnet, 1972; Green and Perkins, 1980). Hence only a selective review of literature, on the elastic analysis of shells, limit state philosophy, and optimization technique and its application to reinforced concrete water towers will be made in this chapter. A brief review of literature for limit analysis of reinforced concrete shells and cracking phenomenon of reinforced concrete under service loads has been given at appropriate places.

2.1 Elastic Analysis of Shells

The shell theories based on linear elasticity concept are most commonly used. These theories adequately predict stresses and deformations for shells exhibiting small deformations. The nonlinear theory of elasticity forms the basis for finite-deflection and stability theories of shells. Large-deflection theories are often required when dealing with shallow shells, highly elastic membranes and buckling problems. The nonlinear shell equations are considerably more difficult to solve and for this reason are more limited in use.

Love (1944) was the first investigator who presented first-order-approximation shell theory based on classical elasticity and gave simplified strain-displacement relations and consequently the constitutive relations.

Second-order-approximation shell theories retain the t/R terms, where 't' is the shell thickness and 'R' is the least radius of curvature of the middle surface, in comparison with unity in the stress resultant equations and in the strain displacement relations. Several authors (Flugge, 1960; Kempner, 1955; Novozhilov, 1959 and Kraus, 1967) have contributed to the development of these theories.

Foregoing theories take into account the flexural behaviour of shells and generally referred to as 'bending' theories of shell. Using these theories solutions for special cases have been found by many investigators(Pflüger, 1961; Salvadori, 1955; Baltrukonis, 1959; Galletly, 1955 and 1960 and Baker and Cline, 1962).

A special case of bending theory of shells in which, in the study of equilibrium of a shell, all moment expressions are neglected, is known as the membrane theory of shells. First contribution to membrane theory were furnished by Lame and Clapeyron early in nineteenth century. They considered symmetric loading on shells of revolution. A number of authors (Novozhilov, 1959; Goldenveiser, 1961 and Timoshenko and Krieger, 1959) have contributed useful material in the development of membrane theory of shells. Using this theory, solutions to innumerable special cases of practical importance

have been worked out by several investigators (Pflüger, 1961; Flugge, 1960; Salvadori, 1955 and Haas, 1962).

Timoshenko (1940) gave analytical membrane solutions for axisymmetrically loaded orthotropic shells of revolution. Baker (1964) derived governing differential equation for orthotropic composite cylinders subjected to axisymmetric loads and obtained results for some special cases of practical importance.

Gerard (1962) presented solutions for orthotropic cylinders under axial compression using linear stability theory. Becker and Gerard (1962) derived small deflection theory for general instability of orthotropic circular cylindrical shells for external pressure, torsion and axial compression. Experimental data were compared with the theoretical results and was found to agree reasonably well.

Elastic analysis and design of reinforced concrete water tanks, with relevant literature, has been given in some textbooks (Green and Perkin, 1980; Gray and Manning, 1973; Krishna and Jain, 1980; Baikov, 1978).

Development of more exact theoretical equations does not necessarily assist in the solution of practical shell problems, because the resulting theoretical equations can only be solved with great difficulty, and even then only for special cases. The experimental approach is also limited because data are not available for every special case. Practical difficulties in both theory and experiment have led to the development and application of applied engineering

methods for the analysis of shells. While these methods are approximate and are valid only under specific conditions, they generally are very useful and give good accuracy for the analysis of practical engineering shell structures.

Arya (1969-70) obtained, using bending theory of shells, numerical data and presented in a convenient tabular form so that the continuity effect at the junctions of a multi-shell structure could be carried out with ease and accuracy. The use of these data was explained through the analysis of an Intze tank. Kameswara Rao (1982) also presented numerical coefficients for rapid calculations of continuity effect in Intze tank.

Baker et al. (1972) have provided a very good collection of membrane and bending solutions which are useful for the analysis of axisymmetrically loaded shells of revolution encountered in engineering practice. The force method of analysis specially suited for such shell structures, also has been dealt in detail.

Kalwar et al. (1983) described the force method of analysis for axisymmetric multishell structures and applied it to the analysis of a conical water tank supported on cylindrical shaft. Adidam et al. (1985) have also used this method for the analysis.

2.2 Limit State Philosophy

Thought was given to limit state philosophy in Soviet Union as early as 1930 and the limit state approach was embodied in the Russian Code in 1954 (Zalesov, 1973).

The question of what constitutes an acceptable risk or probability of failure is very controversial. Society has long demanded and still does, a degree of safety in structures which is out of proportion to the risk it accepts in other areas of life. As this concept of safety is deep rooted in structural design, the notion of probability of failure, even if it is of the order of 10^{-7}, is repulsive to the majority of present day engineers. This is reflected by Baker's statement (1966) that except for absolutely unforeseeable events, a zero probability of failure has to be demanded from the designer. But, to quote Prof. Freudenthal, 'The difference between safe and unsafe design is in the degree of risk considered acceptable, not in the delusion that such a risk can be completely eliminated'.

For the first time a probabilistic approach to the structural safety, taking into account expected variations in load and strength, was introduced in the structural design in the aircraft industry in the late 1940s.

Following the attempts by Freudenthal (1947, 1948) to derive the safety factor from statistical variations of the design parameters, concept of safety was extensively considered by Pugsley (1951) and Johnson (1953).

A composite international body of research workers and practising designers known as European Committee of Concrete, CEB, was set up in 1953 with the object of providing a sound philosophy of design, together with recommendations on the detailed aspects of design based upon experimental and theoretical research. With the inception of CEB much

more serious attention began to be devoted to the entire subject of structural design in reinforced concrete. The limit state philosophy has been adopted and elaborated by the CEB and formed the basis of that Committee's recommendation (CEB, 1964).

Rowe (1965) forwarded the concept of limit state design and stated that this philosophy is simply a restatement, in a logical and rational manner, of the essential concepts of design which has been obtained for many years.

Cornell (1969) proposed a code format for necessary safety factor in the structural design. He suggested to incorporate the measures of uncertainty in predicting the strength and applied force and a measure of degree of safety in determining the safety factor.

Kemp (1973) gave a number of valid reasons why classical reliability theory cannot be applied, in its purest form, to the design of reinforced concrete structures.

According to the International Standards Organization (TC98, 1973), it is not generally required that structures should be designed to withstand effects of exceptional events like wars. In case of certain other events, like accidents or earthquakes, when their frequency is ill-defined, the designer should ensure that the risks associated with such events are limited.

Ang and Cornell (1974) also pointed out that probability of an adverse event or 'failure' is virtually unavoidable and proposed that the failure should be interpreted

with respect to some predefined limit state. Depending upon the limit state under consideration, the concept of failure probability can be applied to both the safety and the performance of structure. Failure probability need not, however, appear explicitly in the routine design equations.

Ellingwood and Ang (1974) developed the methods to illustrate the quantitative analysis of uncertainties and showed the effect of these uncertainties on the level of risk. At the same time risk associated with existing design procedures with reference to reinforced concrete were evaluated which formed the basis for consistent code development.

Following the attempt by Ravindra et al. (1974) to illustrate reliability-based designs of simple structural elements, Moses (1974) considered the reliability of structural system to safety.

Attempts have been made by Galambos et al. (1982) and Ellingwood et al. (1982) to describe the methodologies to develop probability-based load factors and loading combinations for use with common construction material and technologies and to assess structural resistance.

Ellingwood (1982) examined the relative advantages of a number of alternate safety checking formats with regard to their ability to provide uniform reliability in limit state design.

Ellingwood et al. (1983) suggested that explicit provisions to mitigate the effects of abnormal loads should be a part of building codes and standards.

Since code formulation is an evolutionary process, the new provisions seldom reflect abrupt changes from established ones. The International Standards Organisation, aiming at unification of different methods of structural calculations and ensuring safety of structures, has recommended a semi-probabilistic limit state method. On the basis of these recommendations, codes of practice for design of reinforced concrete structures are being revised to conform to the limit state philosophy. BS 5337, CP 110, IS: 456 are among those in which the limit state theory is incorporated and are referred to in the present work.

As the present codes of practice are semi-probabilistic in nature, the future codes of practice have to be modified drastically to incorporate the data that are likely to be available in the near future.

2.3 Optimization Technique and Reinforced Concrete Water Towers

2.3.1 General

The object of a structural design is to achieve not only a safe and a serviceable structure but also the 'best' and 'economical' one. The methods of achieving optimum designs are primarily concerned with arriving at the 'best' structure according to some criterion of structural efficiency. Usual criteria considered for such optimization are the minimum weight/cost, or the maximum stiffness etc.

Excellent survey made by Wastiutynski and Brandt (1963) and Sheu and Prager (1968) provide a complete

historical development of the methods of structural optimization.

Fletcher and Powell (1963) gave a very stable rapidly converging method for unconstrained nonlinear programming problems of optimization. Whereas Fiacco and McCormick (1968) developed sequential unconstrained minimization technique for the solution of constrained nonlinear minimization problem by transforming it into an equivalent unconstrained problem of minimization.

Fox (1971) and Rao (1978) have dealt with the various methods of optimization and given relevant literature survey in the field.

2.3.2 Application to reinforced concrete water towers

In this section, literature mainly related to application of optimization techniques to reinforced concrete water towers has been covered.

Melchers and Rozvany (1970) solved the problem of minimizing the reinforcement in cylindrical reinforced concrete water tanks of a given thickness using Prager-Shield optimality condition. The most important aspect of cracking was ignored by them in the design.

Nielsen (1974) gave a procedure to find the required area of reinforcement at any point of a shell surface using the plastic design method. Thickness of the shell and the distribution of the internal forces were assumed to be known. Depending on the relative magnitude of the bending moments,

twisting moments, and the membrane forces, the optimal pattern of the reinforcement was chosen.

Wilby (1977a) in his extensive literature survey on structural analysis of reinforced concrete tanks has concluded that very little work has been done on the optimization of tanks using either elastic or plastic models.

Selvanathan (1978) has considered the use of different number of columns and spacing of bracings to arrive at an optimal design of the Intze tank for two capacities using nonlinear programming. The total cost of structure, which includes cost of tank, staging and foundation, has been chosen as the objective function. The working stress method is adopted for the design.

Bond (1979) proposed a method, which combines iso-parametric finite element analysis with a nonlinear optimization technique, to search for structural forms resulting in minimum cost reinforced concrete structures. One of the examples considered is that of a conical water tank. The top diameter and thicknesses at top and bottom have been chosen as design variables. The objective function includes the cost of tank, support and the top dome. Alternative method of design, which is given in BS 5337, has been used to govern the resistance to cracking. Two important parameters, one being the reinforcement and its distribution and the other being the diameter of the supporting structure, are not taken as variables in the optimal design process.

Subramanyam (1981) and Adidam and Subramanyam (1982) investigated the optimal design of surface water tanks using

the limit state design philosophy. Radius of the tank, thickness of the tank wall and the reinforcement areas in the two directions are chosen as design variables. Cost of finished concrete, reinforcement and formwork have been taken as the objective function. The permissible surface crack width has been taken as equal to 0.2 mm which seems to be higher for water retaining structures and the affect of seismic forces is ignored in the design. Resistance to cracking due to hoop tension is ascertained by limiting the stress in reinforcement only without assigning any upper limit for direct tensile stress in concrete which may result in excessive tension in concrete, thereby, causing wide cracks in some cases.

Kalwar et al. (1984) considered the optimal design of reinforced concrete water tower consisting of a conical tank supported on cylindrical shaft using limit state design philosophy and a nonlinear optimization technique. The diameter of the supporting shaft, ratio of the top diameter to bottom diameter of the conical reservoir, thicknesses of various elements and the reinforcements in these elements with practicable logical distribution of reinforcement are taken as design variables. The objective function includes the costs of materials, formwork and labour for superstructure. The procedure given in IS: 3370, with a little modification with regard to permissible stress in steel, has been used to provide resistance to cracking. The effect of escalation in unit costs, of materials and formwork, on

optimal configuration and the design variables is also investigated.

The following studies do not make use of any optimization techniques in the strict sense, but consider ways of minimizing cost or weight of reinforced concrete water tanks.

Wilby (1977b, 1978) has considered the cost minimization of polygonal tanks, in stages. In the first stage, for a given capacity of the tank, the surface area is minimized with a view to minimize the cost of the shuttering. Then the thickness of the rectangular plates and reinforcement in them are determined from individual yield-line collapse mechanisms.

Jain et al. (1979) have carried out extensive parametric studies to arrive at optimal designs of Intze tanks. Various capacities, staging heights, bearing capacities of soils and lateral forces due to wind or earthquake are considered for the analysis. Based on these studies, a number of equations for rapid estimation of cost and materials have been presented. For computing the seismic forces the seismic coefficient method has been used instead of the response spectrum method. Nothing has been said about the final design variables for any of the cases considered and about the procedure adopted to arrive at the optimal designs.

Kameswara Rao and Raghavan (1982) considered the optimal design of Intze tanks on shaft. In this study attempt has been made to optimize the weight of the reservoir portion only. This has resulted in a large diameter of the supporting shaft giving an unpleasant configuration of the overall structure which may not be economical in the true sense of the

word. Moreover, wind forces and the continuity effect at joints have been ignored in the design.

2.4 Concluding Remark

It is obvious, from the foregoing literature review, that application of limit analysis to the design of water tanks has hardly been attempted. Optimal design of water towers, with the exception of the Intze tank, has received very little attention although considerable amount of money is spent on the construction of water towers. In a developing country like India where a large number of water towers will have to be constructed to meet the requirement, optimal design of such structures will be very meaningful.

In view of the foregoing considerations optimal design of reinforced concrete water towers has been considered in the current study. The emphasis has been to obtain practically usable optimal cost designs, conforming to the relevant codes of practice, which will remain safe and serviceable throughout their design life.

CHAPTER 3

OPTIMAL DESIGN OF TYPE I WATER TOWERS

3.1 General

Among the various types of reinforced concrete water reservoirs or tanks, cylindrical, conical and Intze tanks are the most commonly used. Reservoirs with the conical walls have a minimum cross-sectional area at the joints with the supporting towers, which makes it possible to efficiently establish the dimension of the supporting towers. Moreover, conical tanks supported on cylindrical shafts are aesthetically pleasing ones and are preferred because of their better architectural expressiveness. In this chapter optimal design of Type I water towers (Figure 1.1) is presented.

3.2 Design Loads

Various loads to be considered for the design of a water tower are dead load, hydrostatic pressure loading, weight of water, wind, and seismic loads.

The values of wind and seismic loads depend upon the geographical location of the place where the tower is going to be situated. India is divided into three wind and five seismic zones depending upon the intensity of wind pressure and seismicity.

3.2.1 Wind loads

IS: 875 specifies the values of basic wind pressures for the various wind zones in India. These basic wind

pressures are nothing but the equivalent static wind pressures which take into account the dynamic head pulsation due to wind gusts. The basic wind pressures for these zones are given as under

Wind zone	Basic wind pressure*
Zone I	1.0 kN/m^2
Zone II	1.5 kN/m^2
Zone III	2.0 kN/m^2

*For heights upto 30 m above the main retarding surface. For heights more than 30 m IS: 875 is to be referred.

IS: 875 also specifies, for structures of various plan shapes other than rectangular ones, the external wind pressure acting on the projected area in the plane perpendicular to the wind, as the product of basic wind pressure and the shape factor. For structures having circular plan shapes, the specified value of shape factor is 0.7. Hence, in calculating wind loads the shape factor is taken as 0.7.

3.2.2 Seismic loads

For the purpose of determining the seismic forces, the country is classified into five zones. IS: 1893 specifies the values of basic seismic coefficients (α_o) and seismic zone factors (F_o) for computing the design values of horizontal seismic coefficient (α_h). Depending on the problem, one of the following two methods may be used for computing the seismic force:

(a) Seismic coefficient method, and

(b) Response spectrum method.

The Table 3.1 gives the values of α_o and F_o for different zones.

Table 3.1 Values of basic seismic coefficients and seismic zone factors for different zones

Zone No.	Method	
	Seismic coefficient method	Response spectrum method
	Basic horizontal seismic coefficient, α_o	Seismic zone factor, F_o
V	0.08	0.40
IV	0.05	0.25
III	0.04	0.20
II	0.02	0.10
I	0.01	0.05

The values of horizontal seismic coefficients (α_h), in the seismic coefficient and response spectrum methods, are given by the following expressions:

(a) In seismic coefficient method

$$\alpha_h = \beta I \alpha_o \tag{3.1}$$

(b) In response spectrum method

$$\alpha_h = \beta I F_o \frac{S_a}{g} \tag{3.2}$$

where

β = a coefficient depending upon the soil-foundation system,

I = a coefficient depending upon the importance of the structure,

α_o = basic seismic coefficient as given in Table 3.1,

F_o = seismic zone factor for average acceleration spectra as given in Table 3.1, and

$\frac{S_a}{g}$ = average acceleration coefficient as read from Figure 3.1 for appropriate natural period and damping of the structure.

In the present study, for finding the wind and seismic forces, the procedures specified in Sections 3.2.1 and 3.2.2 have been adopted. For finding the value of average acceleration coefficient, the damping of reinforced concrete structure is taken as 5%.

3.2.3 Loading combinations and partial safety factors for loads

It is not always necessary to analyse a structure for all the possible loading combinations for arriving at the governing design forces in different parts of a structure. Therefore, it is useful to look critically at the various possible loading combinations, to decide which of these combinations are most likely to govern the design of a particular portion of the structure. Based upon the preliminary studies conducted by the author and the experience of the several designers and investigators in India, Arya

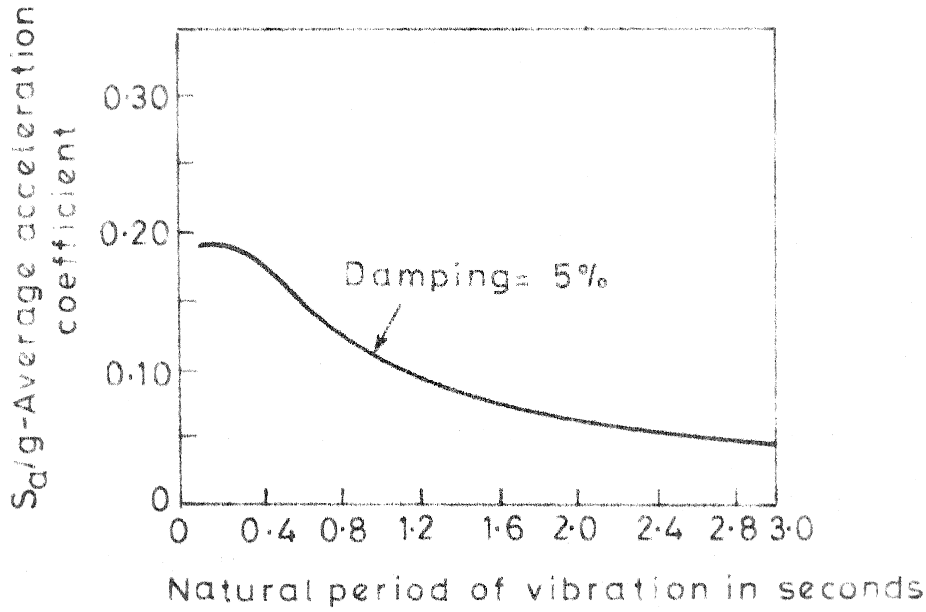


Fig. 3·1 Average acceleration spectra

(1969, 1970), Jain and Singh (1977), Jain et al. (1979), Kameswara Rao (1982) and Kameswara Rao and Raghavan(1982) it is recognised that, for elevated water tanks under Indian environmental conditions, the governing loading combination, for the reservoir portion is the 'dead load + load due to hydrostatic pressure' under tank full condition. Whereas, the loading combination of dead load with wind or seismic forces under full or empty conditions of the reservoir usually govern the design of supporting structure. Depending upon the loading combination and the limit state under consideration, IS: 456 specifies partial safety factors for loads. In the light of the foregoing discussions, the loading combinations and the partial safety factors considered for computing the design loads for various portions of the water tower are given in Table 3.2.

3.3 Elastic Analysis

In order to predict the response of the structure at service loads, elastic analysis is carried out. Water towers considered in the present study can be regarded as multishell structural systems because they are composed of cylindrical, conical and spherical shells and circular rings. These towers are axisymmetric in nature. A shell, in general, is a statically indeterminate structure. Hitherto, thin shell theories based on linear elasticity concepts are most commonly used. The exact solution for a shell structure is very difficult to obtain even for an axisymmetric case. As a result, design engineers derived solutions based upon simple membrane theory.

Table 3.2 Load combinations considered and partial safety factor used for loads

Portion of the tower under consideration	Load combination considered	Partial safety factor					
		Limit states of serviceability			Limit state of collapse		
		DL	LL	WL/EL	DL	LL	WL/EL
Reservoir portion	Dead load (DL) + water pressure under tank full condition (LL)	1.0	1.0	-	1.5	1.5	-
Supporting shaft	(a) Dead load + weight of water under tank full condition (LL)	1.0	1.0	-	1.5	1.5	-
	(b) Dead load + wind/seismic forces under empty tank condition (WL/EL)	1.0	-	1.0	1.5 or 0.9*	-	1.5
	(c) Dead load + wind/seismic forces under tank full condition	1.0	0.8	0.8	1.2	1.2	1.2

*This value is considered when stability against overturning or stress reversal is critical.

However, the solutions so obtained are not found to be satisfactory for most of the practical engineering problems. On the other hand, the bending theory of shells is more general and exact but unfortunately it is highly complex. However, for the class of rotationally symmetric shells subjected to axisymmetric loading, both the theories can be combined to arrive at a near-exact solution. The 'force method' is one such powerful technique. This method of analysis is very well described by Baker et al. (1972). Kalwar et al. (1983) have explained this technique with modified sign conventions, better suited for computer aided analysis, and the same is employed here for finding the design forces in the reservoir portion of the tower. The analysis of the supporting structure is carried out by considering the shaft as a cantilever fixed at the base, subjected to wind or seismic forces under full and empty conditions of the reservoir.

3.3.1 Elastic analysis for finding the design forces in the reservoir portion

As discussed in Section 3.2.3, the governing loading combination for the reservoir portion of the water tower is the dead load + load due to hydrostatic pressure under tank full condition, which conforms to the analysis of a rotationally symmetric multishell structure subjected to axisymmetric loading. Hence, the 'force method' of shell analysis, briefly described in the next section, is used to analyse the tower for finding the design forces in the reservoir portion.

It is worth bringing into light a few facts prior to the introduction of the force method of analysis of axisymmetric shells. When the two solutions, one obtained by simple membrane theory and the other by exact bending theory, are compared, a few useful observations can be made.

(a) The stresses and deformations obtained by the two methods are almost identical for all locations of the shell, with the exception of a narrow strip on the shell surface which is adjacent to the boundary or a junction. This narrow strip is generally no wider than $\sqrt{Rt}$, where R is the radius and t is the thickness of the shell in case of spherical and cylindrical shells but for a conical shell R is the radius of the circle at the larger end.

(b) Except for the strip along the boundary or a junction, all bending moments, twisting moments and transverse shears are negligible, this causes the entire solution to be practically identical to that of the membrane solution.

(c) In case of the built-in edges, the disturbances along the edge are significant. However, the local bending and shear decrease rapidly along the meridian and may become negligible outside the narrow strip described in (a).

3.3.1.1 Force method

The force method of solution for axisymmetric cases can be explained with the aid of a simple example of a spherical

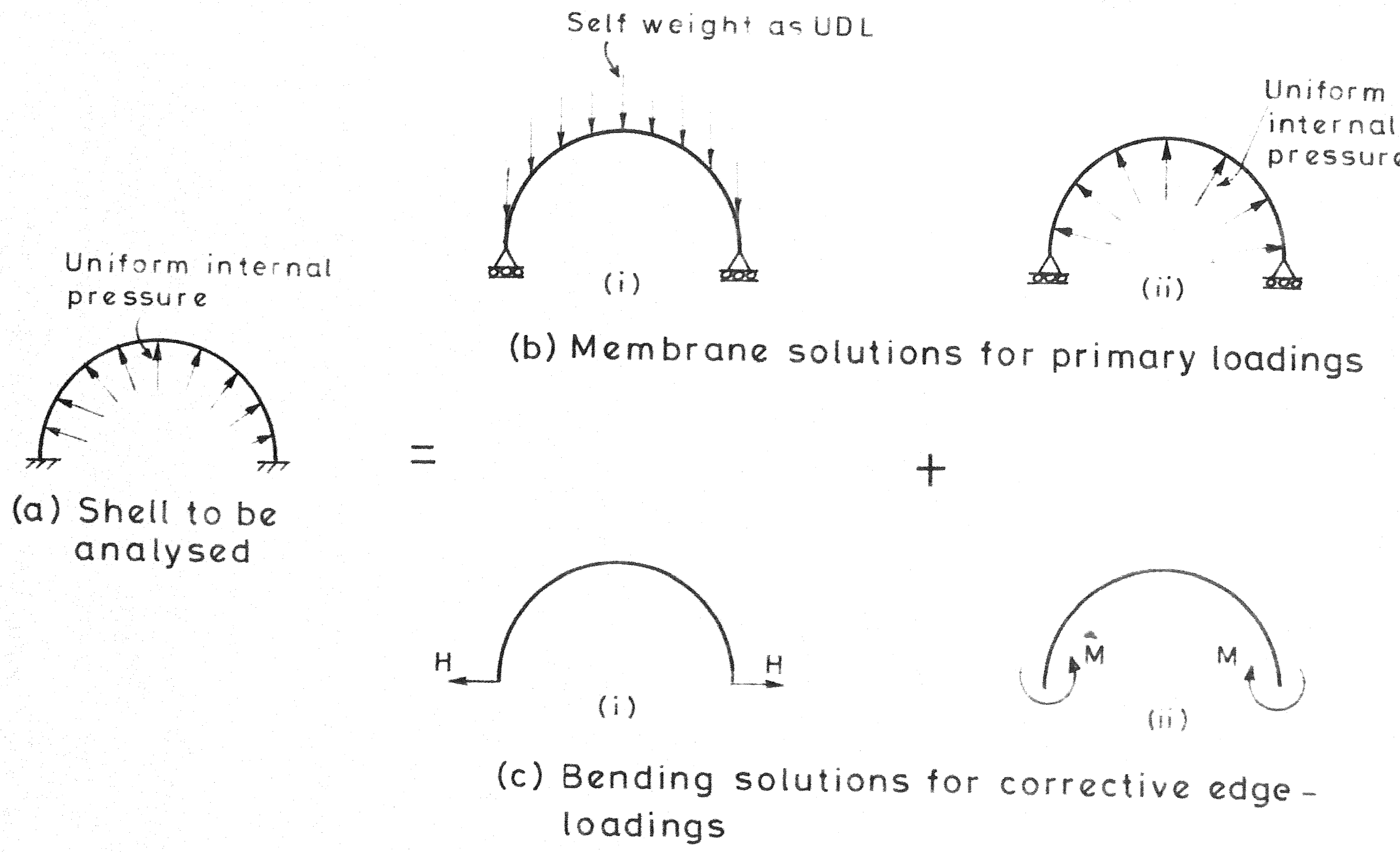


Fig.3·2 Force method of solution

shell with built-in edges subjected to an axisymmetric loading as shown in Figure 3.2(a).

Since the bending and membrane theories give practically the same result, except for a strip adjacent to the boundary, the simple membrane theory can be applied to obtain the solutions for the cases shown in Figure 3.2(b). The Figure 3.2(c) shows the corrective forces, moments and horizontal shears respectively, at the edge which will bring the displaced edge of the shell into the position prescribed by boundary conditions. The amount of edge corrective forces, M and H, will depend upon the magnitude of the deformations caused by the primary loading. The final solution can be obtained by superposing the membrane solutions for primary loadings and the solutions for corrective loadings obtained by bending theory. This method of solving the problem is also known as the unit-edge-force method. This method is quite similar to the force method used to analyse the discrete structural systems.

The force method explained for a simple element, can be extended to multishell problems in a similar manner by blowing the multishell structure into simple shell elements, for which both primary and secondary solutions are readily available, and then impose the equilibrium and compatibility conditions at the junctions to arrive at the solution.

The Figure 3.3 shows the water tower under consideration, whereas the Figure 3.4 shows the same, exploded into simple shell elements, with corrective loadings at various junctions shown along their positive direction. It should be

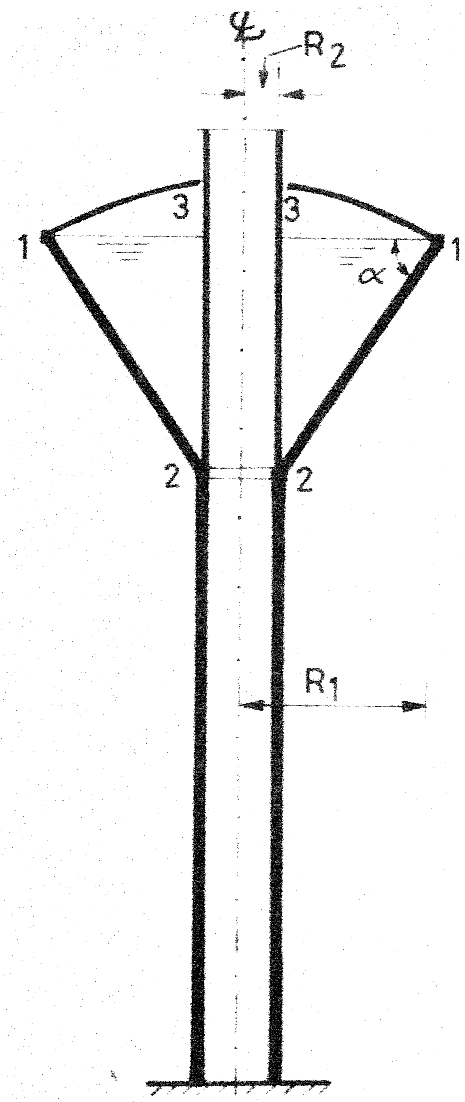


Fig. 3·3 Details of Type I water tower

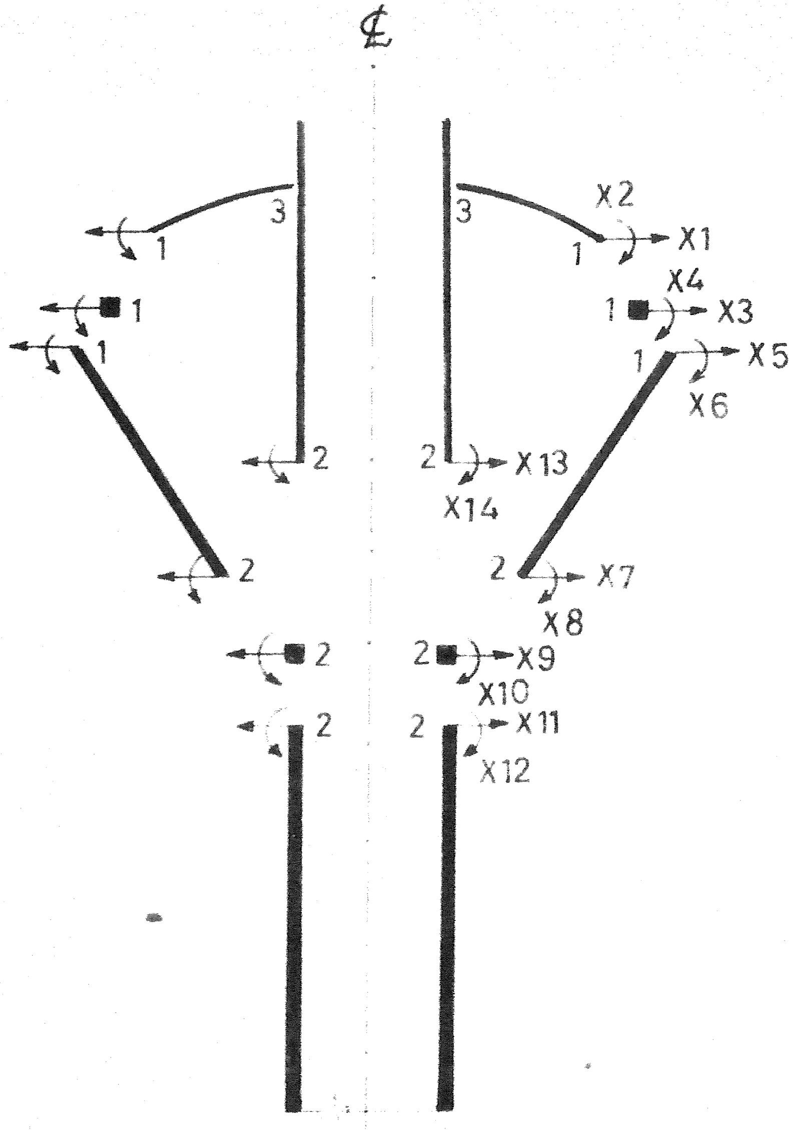


Fig.3·4 The corrective edge loadings at each junction(positive as

noted that at junction 3, the elements are not connected to each other and hence no corrective force appears at this junction. However, even if the elements meeting at this junction are connected rigidly, the amount of secondary forces obtained by actual analysis is found to be negligible. This is because of the fact that the deformations of these elements at this junction, due to primary loading is very small. Moreover, due to the large size of all the elements, the deformations at one end due to secondary forces acting at the other corresponding end of these elements is zero. Therefore, treating the shell elements meeting at this junction as connected or otherwise will not cause any error in the solution of the overall problem. This simplification reduces the size of the problem and also helps considerably in saving the computational time, particularly in the framework of optimum design.

The sign convention for corrective edge-loadings and deformations at junctions, used for solution by force method, can be stated as under.

Right-hand side:

All moments in the clockwise direction,
horizontal loads directed outwardly,
outward displacements, and
clockwise rotations at all the junctions, are considered to be positive.

Left-hand side:

All moments in the anti-clockwise direction,

horizontal loads directed outwardly,

outward displacements, and

anti-clockwise rotations, at all the junctions, are considered positive.

3.3.1.2 Compatibility of deformations and equilibrium conditions

The compatibility of deformations and equilibrium conditions to be satisfied at various junctions are as follows:

Junction 1:

At this junction three elements, top dome, ring beam and conical shell, are rigidly connected to each other. The compatibility of deformations leads us to the fact that rotation and displacement of each element, meeting at this junction, due to the combined effect of various primary and secondary loadings should be the same. Hence the following equations can be written:

$$\delta^{CN1} = \delta^{TD1} \quad \text{or} \quad \delta_P^{CN1} + \delta_S^{CN1} = \delta_P^{TD1} + \delta_S^{TD1} \tag{3.3}$$

and
$$\delta^{CN1} = \delta^{B1} \quad \text{or} \quad \delta_P^{CN1} + \delta_S^{CN1} = \delta_P^{B1} + \delta_S^{B1} \tag{3.4}$$

where δ^{CN1}, δ^{TD1} and δ^{B1} are the horizontal displacements of the conical shell, the top dome and the ring beam respectively at the junction 1; whereas δ_P^{CN1}, δ_P^{TD1} and δ_P^{B1} and δ_S^{CN1}, δ_S^{TD1} and δ_S^{B1} are the horizontal displacements of these elements due to primary and secondary loadings respectively at the

The Eqs. (3.3) and (3.4) can be rewritten as

$$\delta_S^{CN1} - \delta_S^{TD1} = \delta_P^{TD1} - \delta_P^{CN1} \tag{3.5}$$

and

$$\delta_S^{CN1} - \delta_S^{B1} = \delta_P^{B1} - \delta_P^{CN1} \tag{3.6}$$

Similar compatibility equations for rotation at junction 1 can be written as

$$\beta_S^{CN1} - \beta_S^{TD1} = \beta_P^{TD1} - \beta_P^{CN1} \tag{3.7}$$

$$\beta_S^{CN1} - \beta_S^{B1} = \beta_P^{B1} - \beta_P^{CN1} \tag{3.8}$$

where β_P^{CN1}, β_P^{TD1} and β_P^{B1} and β_S^{CN1}, β_S^{TD1} and β_S^{B1} are the rotations of the conical shell, the top dome and the ring beam at the junction 1 due to primary and secondary loadings respectively.

It is important to note that, in this particular case, the size of each element is such that the disturbances caused by the corrective loadings at one end will die at a short distance and will not produce any disturbance at the other end. Hence, the secondary deformations at one edge of an element will be the function of corrective forces acting at that end only and can be expressed as

$$\delta_S^{TD1} = f_{11}^{TD1}\, X1 + f_{12}^{TD1}\, X2 \tag{3.9}$$

$$\beta_S^{TD1} = f_{21}^{TD1}\, X1 + f_{22}^{TD1}\, X2 \tag{3.10}$$

The Eqs. (3.9) and (3.10) can be written in a convenient form using matrix notations as under

$$\begin{bmatrix} \delta_S^{TD1} \\ \beta_S^{TD1} \end{bmatrix} = \begin{bmatrix} f_{11}^{TD1} & f_{12}^{TD1} \\ f_{21}^{TD1} & f_{22}^{TD1} \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} \tag{3.11}$$

where f_{11}^{TD1} = horizontal displacement of the top dome at the junction 1 due to a positive unit horizontal force per unit length acting at that junction,

f_{12}^{TD1} = horizontal displacement of the top dome at the junction 1 due to a positive unit moment per unit length acting at that junction,

f_{21}^{TD1} = rotation of the top dome at the junction 1 due to a positive unit horizontal force per unit length acting at that junction,

and f_{22}^{TD1} = rotation of the top dome at the junction 1 due to a positive unit moment per unit length acting at that junction.

Similar expressions can be written for the ring beam and the conical shell at the junction 1 as under

$$\begin{bmatrix} \delta_S^{B1} \\ \beta_S^{B1} \end{bmatrix} = \begin{bmatrix} f_{11}^{B1} & f_{12}^{B1} \\ f_{21}^{B1} & f_{22}^{B1} \end{bmatrix} \begin{bmatrix} X3 \\ X4 \end{bmatrix} \tag{3.12}$$

and

$$\begin{bmatrix} \delta_S^{CN1} \\ \beta_S^{CN1} \end{bmatrix} = \begin{bmatrix} f_{11}^{CN1} & f_{12}^{CN1} \\ f_{21}^{CN1} & f_{22}^{CN1} \end{bmatrix} \begin{bmatrix} X5 \\ X6 \end{bmatrix} \tag{3.13}$$

The equilibrium conditions will yield two more equations for the junction 1 as follows

$$X1 + X3 + X5 = 0 \tag{3.14}$$

$$X2 + X4 + X6 = 0 \tag{3.15}$$

The relations (3.11) through (3.13), the compatibility conditions (3.5) through (3.8) together with the equilibrium Eqs. (3.14) and (3.15) can be arranged as a set of six linear algebraic equations in six unknowns (X1 to X6) as under

$$\begin{bmatrix} -f_{11}^{TD1} & -f_{12}^{TD1} & 0 & 0 & f_{11}^{CN1} & f_{12}^{CN1} \\ -f_{21}^{TD1} & -f_{22}^{TD1} & 0 & 0 & f_{21}^{CN1} & f_{22}^{CN1} \\ 0 & 0 & -f_{11}^{B1} & -f_{12}^{B1} & f_{11}^{CN1} & f_{12}^{CN1} \\ 0 & 0 & -f_{21}^{B1} & -f_{22}^{B1} & f_{21}^{CN1} & f_{22}^{CN1} \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ X4 \\ X5 \\ X6 \end{bmatrix} = \begin{bmatrix} \delta_P^{TD1} - \delta_P^{CN1} \\ \beta_P^{TD1} - \beta_P^{CN1} \\ \delta_P^{B1} - \delta_P^{CN1} \\ \beta_P^{B1} - \beta_P^{CN1} \\ 0 \\ 0 \end{bmatrix} \tag{3.16}$$

Junction 2:

At this junction four elements, the ring beam, the conical shell, the supporting cylindrical shell and the inner cylindrical shell are rigidly connected to each other. Proceeding in the same way, the compatibility and equilibrium conditions can be expressed by a relation similar to that given by Eq. (3.16) for the junction 2. This relation will represent another set of eight linear equations with eight unknowns (X7 to X14) as follows.

$$\begin{bmatrix} f_{11}^{CN2} & f_{12}^{CN2} & -f_{11}^{B2} & -f_{12}^{B2} & 0 & 0 & 0 & 0 \\ f_{21}^{CN2} & f_{22}^{CN2} & -f_{21}^{B2} & -f_{22}^{B2} & 0 & 0 & 0 & 0 \\ f_{11}^{CN2} & f_{12}^{CN2} & 0 & 0 & -f_{11}^{S2} & -f_{12}^{S2} & 0 & 0 \\ f_{21}^{CN2} & f_{22}^{CN2} & 0 & 0 & -f_{21}^{S2} & -f_{22}^{S2} & 0 & 0 \\ f_{11}^{CN2} & f_{12}^{CN2} & 0 & 0 & 0 & 0 & -f_{11}^{CY2} & -f_{12}^{CY2} \\ f_{21}^{CN2} & f_{22}^{CN2} & 0 & 0 & 0 & 0 & -f_{21}^{CY2} & -f_{22}^{CY2} \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} X7 \\ X8 \\ X9 \\ X10 \\ X11 \\ X12 \\ X13 \\ X14 \end{bmatrix} = \begin{bmatrix} \delta_P^{B2} - \delta_P^{CN2} \\ \beta_P^{B2} - \beta_P^{CN2} \\ \delta_P^{S2} - \delta_P^{CN2} \\ \beta_P^{S2} - \beta_P^{CN2} \\ \delta_P^{CY2} - \delta_P^{CN2} \\ \beta_P^{CY2} - \beta_P^{CN2} \\ 0 \\ 0 \end{bmatrix} \quad (3.1$$

where δ_p^{CN2}, δ_p^{B2}, δ_p^{S2} and δ_p^{CY2} and β_p^{CN2}, β_p^{B2}, β_p^{S2} and β_p^{CY2} are the displacements and rotations at junction 2, of conical shell, ring beam, supporting shaft and inner cylindrical shell respectively, due to primary loadings

f_{11}^{CN2} = horizontal displacement of the conical shell at the junction 2 due to a positive unit horizontal force per unit length acting at that junction,

f_{12}^{CN2} = horizontal displacement of the conical shell at the junction 2 due to a positive unit moment per unit length acting at that junction,

f_{21}^{CN2} = rotation of the conical shell at the junction 2 due to a positive unit horizontal force per unit length acting at that junction, and

f_{22}^{CN2} = rotation of the conical shell at the junction 2 due to a positive unit moment per unit length acting at that junction.

Similarly f_{11}^{B2}, f_{12}^{B2}, f_{21}^{B2}, f_{22}^{B2}, f_{11}^{S2}, f_{12}^{S2}, f_{21}^{S2}, f_{22}^{S2}, f_{11}^{CY2}, f_{12}^{CY2}, f_{21}^{CY2} and f_{22}^{CY2} represent the displacements and rotations at junction 2 due to unit edge loadings for ring beam, cylindrical support, and the inner cylinder respectively.

As already discussed earlier, the size of all the shell elements in this particular case are such that the influence of the corrective loadings at one end do not interact with those caused by the similar loading at the other end. Hence, the two sets of equations, (3.16) and (3.17), can be solved independently or combined together to form a single set of fourteen linear algebraic equations in fourteen

unknowns to arrive at the solution. In certain cases, where the stresses and deformations at one end are influenced by the corrective-edge loadings at the other end, the solution can be obtained only by solving the combined set of linear relations involving all the unknowns.

When Eqs. (3.16) and (3.17) are combined together, the resulting equation will be of the form as follows

$$[f] \quad \bar{X} \quad = \quad \bar{\Delta}_P \tag{3.18}$$

in which $[f]$ is the flexibility matrix,

$\bar{X}$ is the load vector of the unknown corrective edge-loading, and

$\bar{\Delta}_P$ is the deformation vector due to primary loading.

It can be seen from Eq. (3.18) that the solution by force method requires formulae for obtaining the elements of the flexibility matrix and the deformations of the various shell elements at the junctions due to primary loadings.

Most of the expressions for stress resultants and deformations at any point of the shell, needed for the present work, are documented in the existing literature. A few such expressions, for specific cases, are derived in the present work. All these are listed in tabular form for various primary and edge-loadings. These expressions are given for both special and general cases to obtain deformations at junctions only and the stress resultants and deformations at any point of the shell respectively. For primary loading

solutions are obtained by using membrane theory whereas bending theory is used for the edge-loading.

3.3.1.3 Special solutions for deformations at junctions

These expressions are nothing but a special case of the general load-deformation relations at any point of the shell. The Tables 3.4 through 3.10 give the formulae for deformations at junctions due to various primary and corrective edge-loadings. The sign-conventions adopted here is the same as mentioned in Section 3.3.1.1. In order to put the expressions for the bending solution in a convenient form for conical and spherical shells, factors F_i and $F_i(n)$ are used. These factors are given in the Table 3.3 in which the symbols K, L and n for conical and spherical shells are given as follows.

Conical shell:

$$K = \frac{\sqrt[4]{[3(1 - \mu^2)]}}{\sqrt{(t\ x_m \cot \alpha)}}, \qquad n = \frac{\bar{x}}{\bar{L}}$$

and

$$D = \frac{Et^3}{12(1 - \mu^2)}$$

where t = thickness of the shell,

μ = Poisson's ratio,

E = Young's modulus of elasticity, and

L, x_m, $\bar{x}$, $\bar{L}$ and α are as indicated in Figure 3.5.

Spherical shell:

$$K = \sqrt[4]{[3(1 - \mu^2)(\tfrac{R}{t})^2]}, \qquad L = \phi_1 - \phi_2$$

Table 3.3 $F_i(n)$ and F_i factors

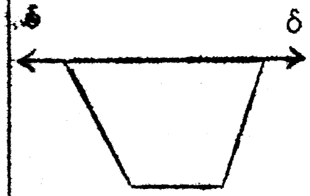
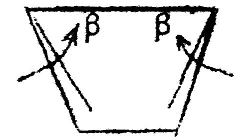
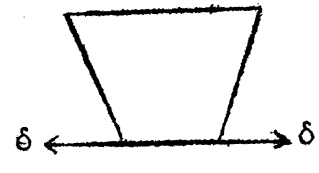
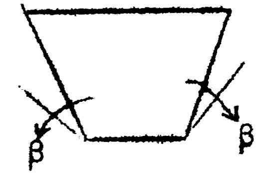
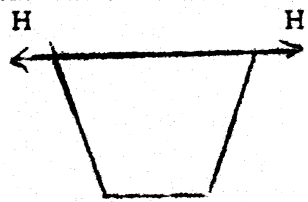
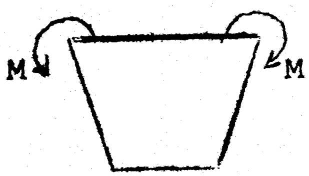
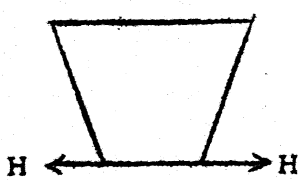
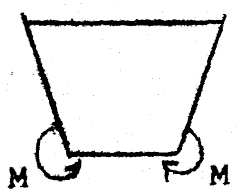
i	$F_i(n)$	F_i
1	$\sinh^2 kLn - \sin^2 kLn$	$\sinh^2 kL - \sin^2 kL$
2	$\sinh^2 kLn + \sin^2 kLn$	$\sinh^2 kL + \sin^2 kL$
3	$\sinh kLn \cosh kLn + \sin kLn \cos kLn$	$\sinh kL \cosh kL + \sin kL \cos kL$
4	$\sinh kLn \cosh kLn - \sin kLn \cos kLn$	$\sinh kL \cosh kL - \sin kL \cos kL$
5	$\sin^2 kLn$	$\sin^2 kL$
6	$\sinh^2 kLn$	$\sinh^2 kL$
7	$\cosh kLn \cos kLn$	$\cosh kL \cos kL$
8	$\sinh kLn \sin kLn$	$\sinh kL \sin kL$
9	$\cosh kLn \sin kLn - \sinh kLn \cos kLn$	$\cosh kL \sin kL - \sinh kL \cos kL$
10	$\cosh kLn \sin kLn + \sinh kLn \cos kLn$	$\cosh kL \sin kL + \sinh kL \cos kL$
11	$\sin kLn \cos kLn$	$\sin kL \cos kL$
12	$\sinh kLn \cosh kLn$	$\sinh kL \cosh kL$
13	$\cosh kLn \cos kLn - \sinh kLn \sin kLn$	$\cosh kL \cos kL - \sinh kL \sin kL$
14	$\cosh kLn \cos kLn + \sinh kLn \sin kLn$	$\cosh kL \cos kL + \sinh kL \sin kL$

Contd...

Table 3.3 (continued)

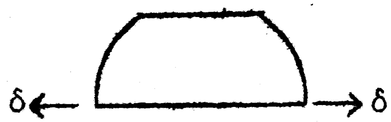
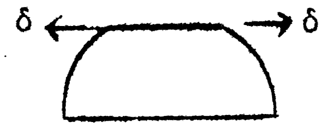
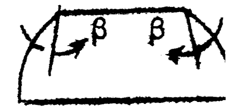
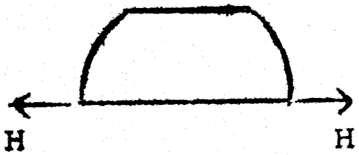
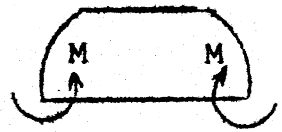
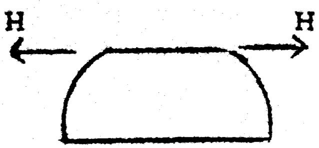
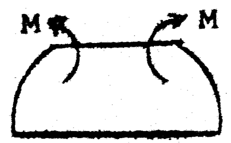
i	$F_i(n)$	F_i
15	$\cosh kLn \sin kLn$	$\cosh kL \sin kL$
16	$\sinh kLn \cos kLn$	$\sinh kL \cos kL$
17	$\exp(-kLn \cos kLn)$	$\exp(-kL \cos kL)$
18	$\exp(-kLn \sin kLn)$	$\exp(-kL \sin kL)$
19	$\exp[-kLn(\cos kLn + \sin kLn)]$	$\exp[-kL(\cos kL + \sin kL)]$
20	$\exp[-kLn(\cos kLn - \sin kLn)]$	$\exp[-kL(\cos kL - \sin kL)]$

Table 3.4 Open conical shell, special solutions for deformations at junctions due to edge-loadings

Edge-deformation / Loading condition				
	$\frac{H}{2DK^3}(\frac{F_4}{F_1})\sin^2\alpha$	$\frac{H}{2DK^2}(\frac{F_2}{F_1})\sin\alpha$	$-\frac{H}{2DK^3}(\frac{F_9}{F_1})\sin^2\alpha$	$\frac{H}{DK^2}(\frac{F_8}{F_1})\sin\alpha$
	$\frac{M}{2DK^2}(\frac{F_2}{F_1})\sin\alpha$	$\frac{M}{DK}(\frac{F_3}{F_1})$	$-\frac{M}{DK^2}(\frac{F_8}{F_1})\sin\alpha$	$\frac{M}{DK}(\frac{F_{10}}{F_1})$
	$-\frac{H}{2DK^3}(\frac{F_9}{F_1})\sin^2\alpha$	$-\frac{H}{DK^2}(\frac{F_8}{F_1})\sin\alpha$	$\frac{H}{2DK^3}(\frac{F_4}{F_1})\sin^2\alpha$	$-\frac{H}{2DK^2}(\frac{F_2}{F_1})\sin\alpha$
	$\frac{M}{DK^2}(\frac{F_8}{F_1})\sin\alpha$	$\frac{M}{DK}(\frac{F_{10}}{F_1})$	$-\frac{M}{2DK^2}(\frac{F_2}{F_1})\sin\alpha$	$\frac{M}{DK}(\frac{F_3}{F_1})$

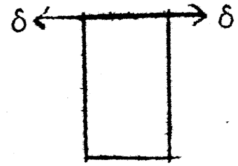
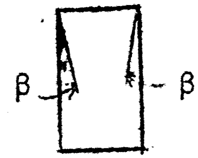
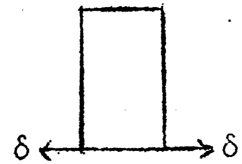
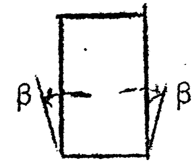
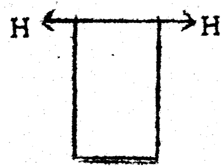
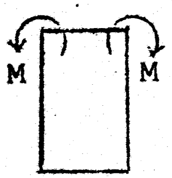
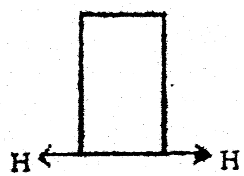
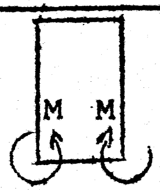
For F_1, D and K see Table 3.3 and Section 3.3.1.3.

Table 3.5 Open spherical shell, special solutions for deformations at junctions due to edge-loadings

Edge deformation / Loading condition				
	$-\frac{2HRK}{Et}\left(\frac{F_4}{F_1}\right)\sin^2\phi_1$	$-\frac{2HK^2}{Et}\left(\frac{F_2}{F_1}\right)\sin\phi_1$	$-\frac{2HRK}{Et}\left(\frac{F_9}{F_1}\right)\sin\phi_1\ \sin\phi_2$	$-\frac{4HK^2}{Et}\left(\frac{F_8}{F_1}\right)$
	$-\frac{2MK^2}{Et}\left(\frac{F_2}{F_1}\right)\sin\phi_1$	$\frac{4MK^3}{EtR}\left(\frac{F_3}{F_1}\right)$	$\frac{4MK^2}{Et}\left(\frac{F_8}{F_1}\right)\sin\phi_2$	$\frac{4MK^3}{EtR}\left(\frac{F_{10}}{F_1}\right)$
	$-\frac{2HRK}{Et}\left(\frac{F_9}{F_1}\right)\sin\phi_1\ \sin\phi_2$	$\frac{4HK^2}{Et}\left(\frac{F_8}{F_1}\right)\sin\phi_2$	$\frac{2HRK}{Et}\left(\frac{F_4}{F_1}\right)\sin^2\phi_2$	$\frac{2HK^2}{Et}\left(\frac{F_2}{F_1}\right)\sin$
	$-\frac{4MK^2}{Et}\left(\frac{F_8}{F_1}\right)\sin\phi_1$	$-\frac{4MK^3}{EtR}\left(\frac{F_{10}}{F_1}\right)$	$\frac{2MK^2}{Et}\left(\frac{F_2}{F_1}\right)\sin\phi_2$	$\frac{4MK^3}{EtR}\left(\frac{F_3}{F_1}\right)$

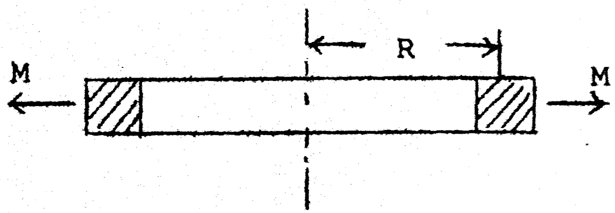
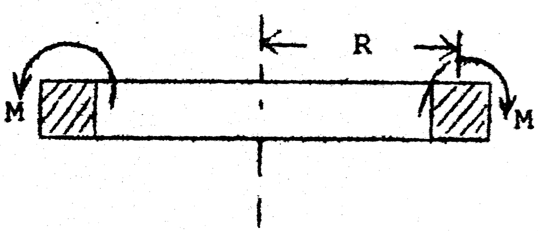
For F_1 and K see Table 3.3 and Section 3.3.1.3.

Table 3.6 Long cylindrical shell, special solutions for deformations at junctions due to edge loadings

Edge deformation / Loading condition				
	$\frac{HL^3}{2DK_1^3}$	$\frac{HL^2}{2DK_1^2}$	0	0
	$\frac{ML^2}{2DK_1^2}$	$\frac{ML}{DK_1}$	0	0
	0	0	$\frac{HL^3}{2DK_1^3}$	$-\frac{HL^2}{2DK_1^2}$
	0	0	$-\frac{ML^2}{2DK_1^2}$	$\frac{ML}{DK_1}$

For D and K_1 see Section 3.3.1.3.

Table 3.7 Circular ring, solutions for deformations due to edge-loadings

Loading condition \ Deformation	Outward radial displacement δ	Clockwise rotation β
	$\frac{HR^2}{AE}$	0
	0	$\frac{MR^2}{EI}$

where A = area of cross section of the ring

I = moment of inertia of the cross section of the ring about the centroidal axis in the plane of th ring.

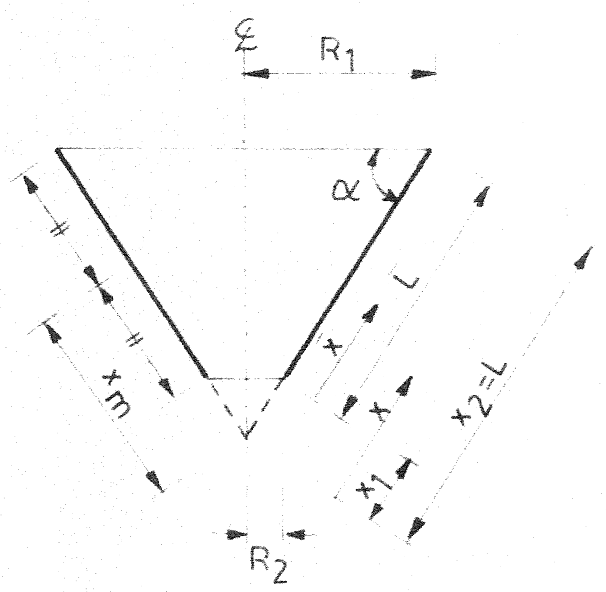


Fig. 3·5 Details of conical shell

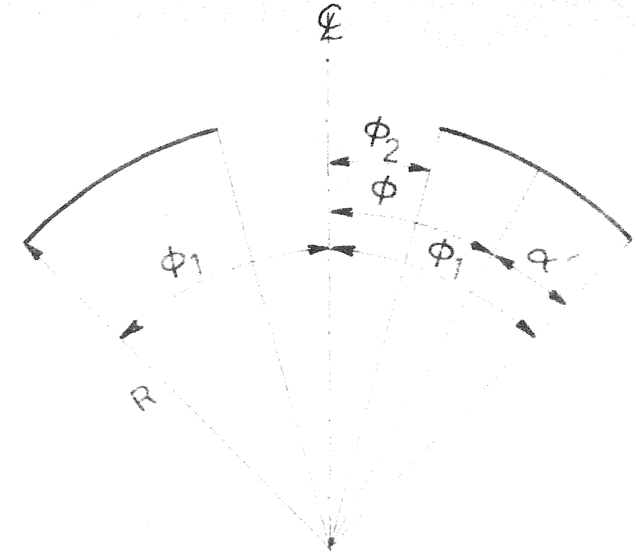


Fig. 3·6 Details of spherical shell

Fig. 3·7 Details of cylindrical shell

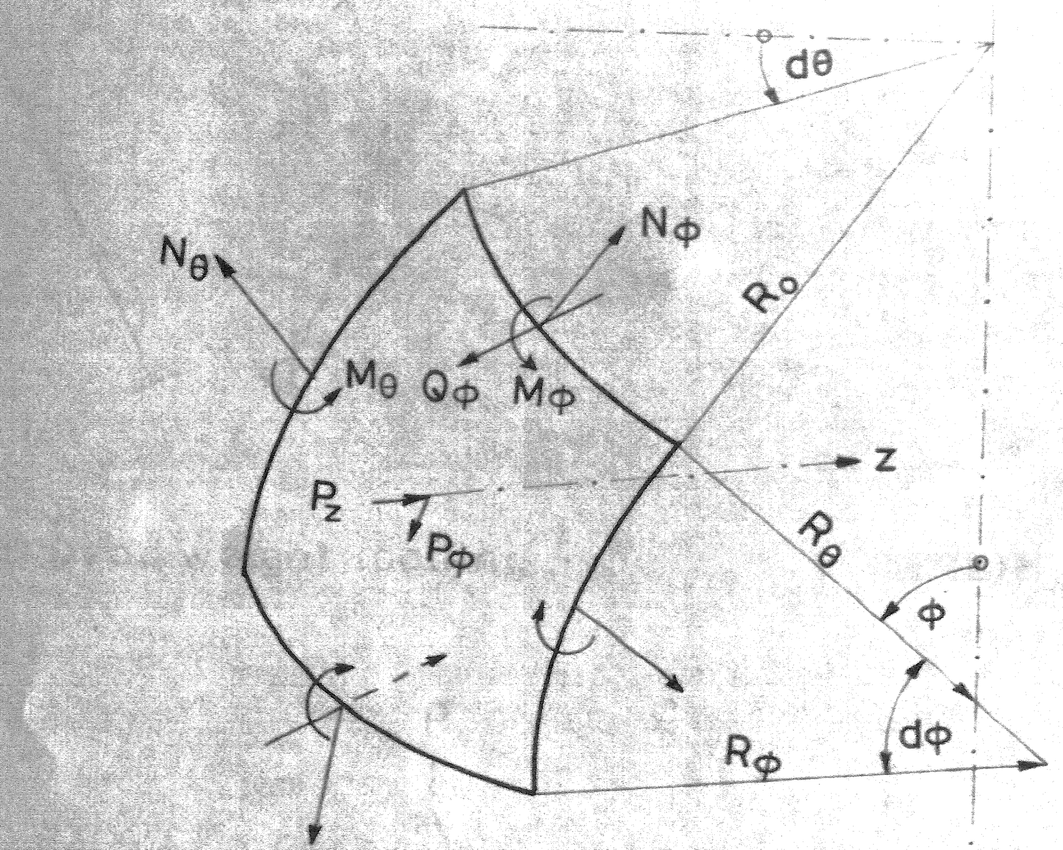
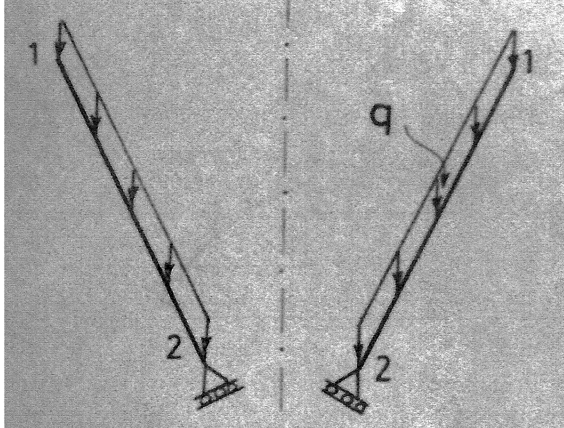
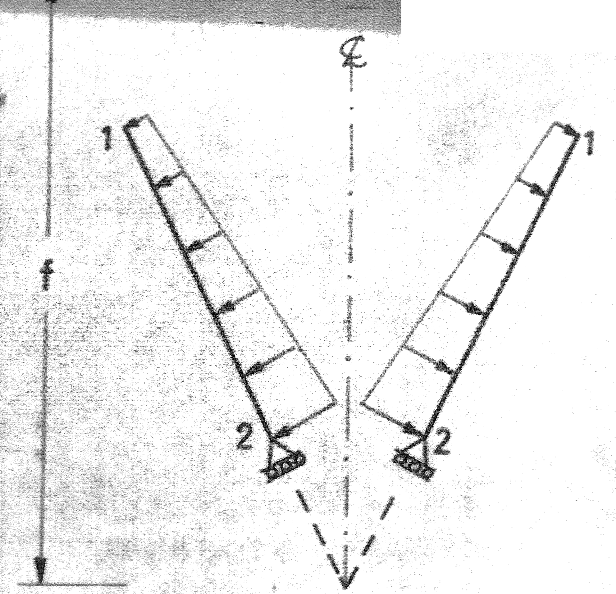


Fig. 3·8 An infinitesimal element of an axisymmetrically loaded shell of revolution (stress resultants positive as shown)

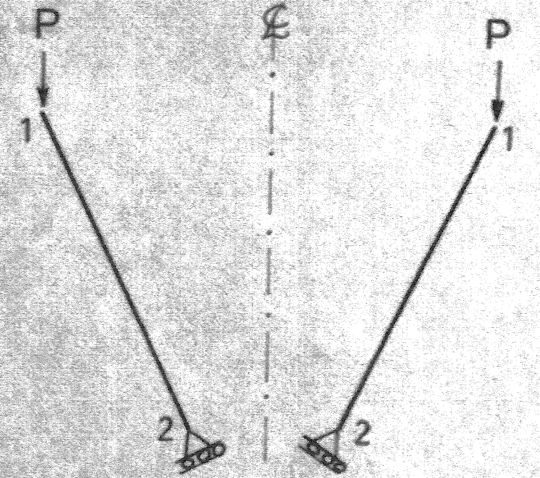


(a) Dead weight loading

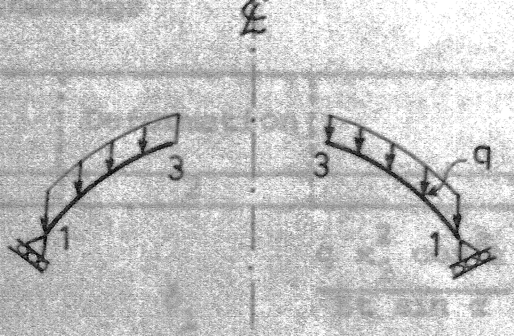


(b) Hydrostatic pressure loading

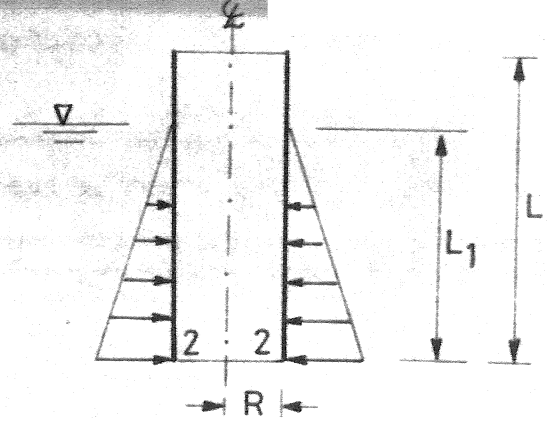
Note: 'f' for present case is equal to x_2 Sin α (Fig. 3·5)



(c) Equally distributed loading 'P' per unit



(a) Spherical shell — dead weight loading



(b) Cylindrical shell — hydrostat[ic] pressure loading

Fig.3·10 Primary loadings on spherical and cylindrical shells

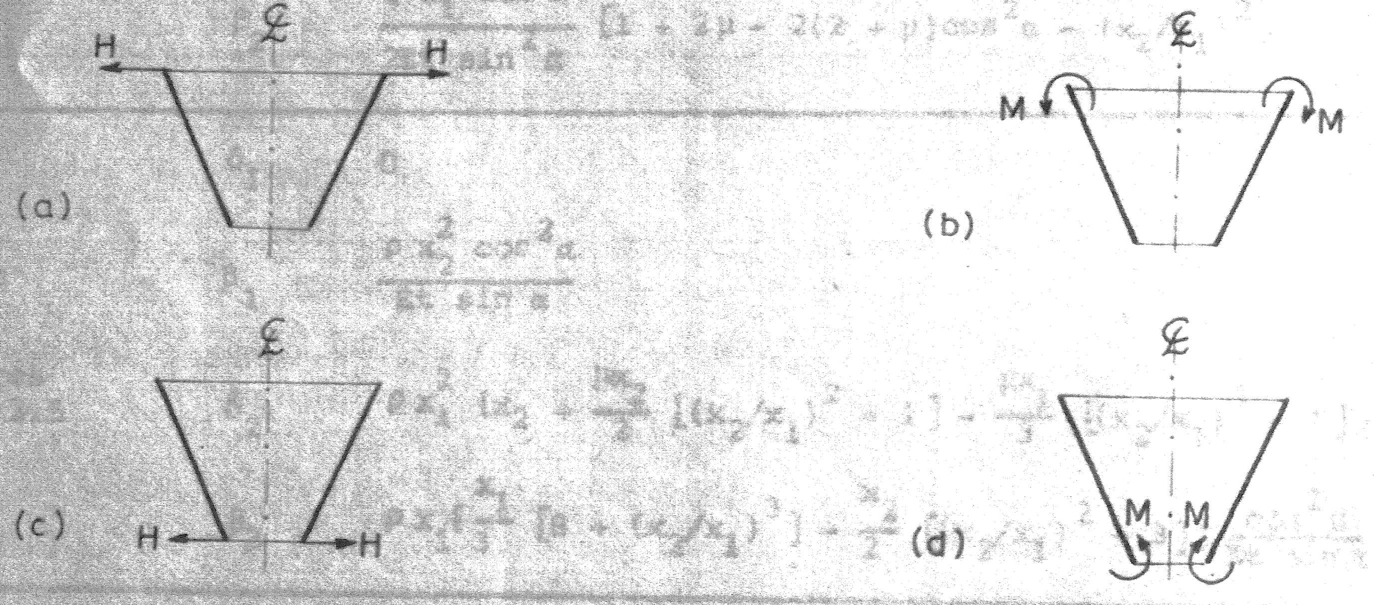


Table 3.8 Open conical shell, special solutions for deformations at junctions due to primary loadings

Loading condition	Deformation	Algebraic expression for deformation
1	2	3
Dead weight Refer Figures 3.9(a) and 3.5	δ_1	$\dfrac{q\, x_2^2 \cos^3\alpha}{Et \sin\alpha}$
	β_1	$\dfrac{q\, x_2 \cos\alpha}{Et \sin^2\alpha}\,[\mu - (2+\mu)\cos^2\alpha]$
	δ_2	$\dfrac{q\, x_1^2 \cot\alpha}{2Et}\,[2\cos^2\alpha + \mu\,(x_2/x_1)^2 - \mu]$
	β_2	$\dfrac{q\, x_1 \cos\alpha}{2Et \sin^2\alpha}\,[1 + 2\mu - 2(2+\mu)\cos^2\alpha - (x_2/x_1)^2]$
Hydrostatic pressure loading. Refer Figures 3.9(b) and 3.5	δ_1	0
	β_1	$\dfrac{\rho\, x_2^2 \cos^2\alpha}{Et \sin\alpha}$
	δ_2	$\rho\, x_1^2 \{x_2 + \dfrac{\mu x_2}{2}[(x_2/x_1)^2 - 1] - \dfrac{\mu x_1}{3}[(x_2/x_1)^3 - 1] - x_1\}\dfrac{\cos^2\alpha}{Et}$
	β_2	$\rho\, x_1\{\dfrac{x_1}{3}[8 + (x_2/x_1)^3] - \dfrac{x_2}{2}[(x_2/x_1)^2 + 3]\}\dfrac{\cos^2\alpha}{Et \sin\alpha}$

Contd...

Table 3.8 (continued)

1	2	3
Equally distributed loading along the edge. Refer Figures 3.9(c) and 3.5	δ_1	$\frac{P \mu x_2 \cot \alpha}{Et}$
	β_1	$-\frac{P \cot \alpha}{Et \sin \alpha}$
	δ_2	$\frac{P \mu x_2 \cot \alpha}{Et}$
	β_2	$-\frac{P x_2 \cot \alpha}{Et x_1 \sin \alpha}$

where q = weight of shell per unit its surface area,

ρ = unit weight of water,

P = equally distributed load per unit length along the edge.

Note:- 'f' for the present case is equal to $x_2 \sin \alpha$.

Table 3.9 Open spherical shell, special solutions for deformations at junctions due to primary loadings

Loading condition	Deformation	Algebraic expression for deformation
	δ_1	$\frac{R^2 q}{Et} \left[- \cos \phi_1 + \frac{1 + \mu}{\sin^2 \phi_1} (\cos \phi_2 - \cos \phi_1)\right] \sin \phi_1$
Dead weight Refer Figures 3.10(a) and 3.6	β_1	$- \frac{Rq}{Et} (2 + \mu) \sin \phi_1$
	δ_3	$- \frac{R^2 q}{Et} \sin \phi_2 \cos \phi_2$
	β_3	$- \frac{Rq}{Et} (2 + \mu) \sin \phi_2$

where q = weight of shell per unit its surface area.

Table 3.10 Cylindrical shell, special solutions for deformations at junctions due to primary loadings

Loading condition	Deformation	Algebraic expression for deformation
Hydrostatic pressure loading. Refer Figure 3.10(b)	δ_2	$-\dfrac{\rho R^2 L_1}{Et}$
	β_2	$-\dfrac{\rho R^2}{Et}$

where ρ = unit weight of water.

and $\quad n = \dfrac{\alpha'}{\phi_1 - \phi_2}$

where ϕ_1, ϕ_2, R, and α' are as shown in Figure 3.6.

Cylindrical shell:

The corresponding symbols used for cylindrical shell are K_1, $\bar{n}$ and D which are given as under:

$$K_1 = \frac{L}{\sqrt{(Rt)}} \sqrt[4]{[3(1 - \mu^2)]}, \qquad \bar{n} = \frac{x}{L}$$

and $\quad D = \dfrac{Et^3}{12(1 - \mu^2)}$

where x, L, and R are as defined in Figure 3.7.

In the present study both, supporting and the inner cylinder, come under the category of long cylinders in which case $K_1 \geq 5$. Therefore, the simplified solutions which are meant for long cylinders are given for use.

Now using the solutions given in Tables 3.4 through 3.7 for H = M = 1, the various elements of the flexibility matrix [f] can be found. Similarly Tables 3.8 through 3.10 can be used to obtain the deformations of shell elements at various junctions due to primary loadings and then all the elements of deformation vector $\bar{\Delta}_p$ can be determined. The Eq. (3.18) can now be solved to obtain the unknown values of corrective edge-loadings (X1 to X14).

After obtaining the values of X1 to X14, the stresses and deformations at any point of the shell can be obtained by superposing the membrane solutions for primary loadings and the bending solutions for the edge-loadings. For this,

general formulae are needed for both stresses and deformations due to primary and edge-loadings. The next section deals with these general solutions.

3.3.1.4 General solutions for stress resultants and deformations

The Tables 3.11 through 3.16 give solutions for stress resultants and deformations at any point of the shell under consideration for various cases of primary and edge-loadings. The Figure 3.8 shows an infinitesimal element of an axisymmetrically loaded shell of revolution. The element is in equilibrium under the indicated loadings. The stress resultants N_ϕ, N_θ, M_ϕ, M_θ and Q_ϕ shown in Figure 3.8 are taken as positive and all the general solutions given in Tables 3.11 through 3.16 conform to this sign convention. The nomenclature used for different stress resultants and the sign convention mentioned in the foregoing can be stated as under:

N_ϕ = force in the meridional direction, kN/m, positive if causing tension,

N_θ = force in the circumferential direction, kN/m, positive if causing tension,

M_ϕ = moment in the meridional direction, kNm/m, positive if causing tension on inside face,

M_θ = moment in the circumferential direction, kNm/m, positive if causing tension on inside face, and

Q_ϕ = transverse shear force in the meridional direction, kN/m, positive as indicated in Figure 3.8.

Table 3.11 Bending solutions for open conical shell - edge-loadings

Loading condition	Stress resultants and deformations	Algebraic expressions for stress resultants and deformations
1	2	3
Refer Figures 3.11(a) and 3.5	N_x	$H[-(F_9/F_1)\ F_{10}(n) + 2(F_8/F_1)\ F_8(n)] \cos\alpha$
	N_θ	$2H\ x_m K[-(F_9/F_1)\ F_7(n) + (F_8/F_1)\ F_{10}(n)] \cos\alpha$
	M_x	$H[(F_9/F_1)\ F_8(n) - (F_8/F_1)\ F_9(n)]\ (\sin\alpha)/K$
	Q_x	$H[-(F_9/F_1)\ F_{10}(n) + 2(F_8/F_1)\ F_8(n)] \sin\alpha$
	δ	$H[-(F_9/F_1)\ F_7(n) + (F_8/F_1)\ F_{10}(n)]\ (\sin^2\alpha)/(2DK^3)$
	β	$H[(F_9/F_1)\ F_9(n) + 2(F_8/F_1)\ F_7(n)]\ (\sin\alpha/2DK^2)$
Refer Figures 3.11(b) and 3.5	N_x	$-2MK[(F_8/F_1)\ F_{10}(n) - (F_{10}/F_1)\ F_8(n)] \cot\alpha$
	N_θ	$-2MK^2\ x_m[2(F_8/F_1)\ F_7(n) - (F_{10}/F_1)\ F_{10}(n)] \cot\alpha$
	M_x	$M[2(F_8/F_1)\ F_8(n) - (F_{10}/F_1)\ F_9(n)]$
	Q_x	$-2MK[(F_8/F_1)\ F_{10}(n) - (F_{10}/F_1)\ F_8(n)]$
	δ	$-M[2(F_8/F_1)\ F_7(n) - (F_{10}/F_1)\ F_{10}(n)]\ (\sin\alpha)/(2DK^2)$
	β	$M[(F_8/F_1)\ F_9(n) + (F_{10}/F_1)\ F_7(n)]/(DK)$

Contd...

Table 3.11 (continued)

1	2	3
	N_x	$-H[F_7(n) - (F_4/F_1)F_{10}(n) + (F_2/F_1)F_8(n)]\cos\alpha$
	N_θ	$H x_m K[F_9(n) + 2(F_4/F_1)F_7(n) - (F_2/F_1)F_{10}(n)]\cos\alpha$
Refer Figures 3.11(c) and 3.5	M_x	$H[F_{10}(n) - 2(F_4/F_1)F_8(n) + (F_2/F_1)F_9(n)](\sin\alpha)/(2K)$
	Q_x	$-H[F_7(n) - (F_4/F_1)F_{10}(n) + (F_2/F_1)F_8(n)]\sin\alpha$
	δ	$H[F_9(n) + 2(F_4/F_1)F_7(n) - (F_2/F_1)F_{10}(n)](\sin^2\alpha)/(4DK^3)$
	β	$-H[-F_8(n) + (F_4/F_1)F_9(n) + (F_2/F_1)F_7(n)](\sin\alpha)/(2DK$
	N_x	$-2MK[(F_6/F_1)F_{15}(n) + (F_5/F_1)F_{16}(n) - (F_3/F_1)F_8(n)]$ co
	N_θ	$-2M x_m K^2[(F_6/F_1)F_{14}(n) + (F_5/F_1)F_{13}(n) - (F_3/F_1)F_{10}(n)]$c
Refer Figures 3.11(d) and 3.5	M_x	$-M[(F_6/F_1)F_{13}(n) - (F_5/F_1)F_{14}(n) + (F_3/F_1)F_9(n)]$
	Q_x	$-2MK[(F_6/F_1)F_{15}(n) + (F_5/F_1)F_{16}(n) - (F_3/F_1)F_8(n)]$
	δ	$-M[(F_6/F_1)F_{14}(n) + (F_5/F_1)F_{13}(n) - (F_3/F_1)F_{10}(n)](\sin\alpha)/($
	β	$-M[(F_6/F_1)F_{16}(n) - (F_5/F_1)F_{15}(n) - (F_3/F_1)F_7(n)]/(DK)$

For F_i, $F_i(n)$, D and K see Table 3.3 and Section 3.3.1.3.

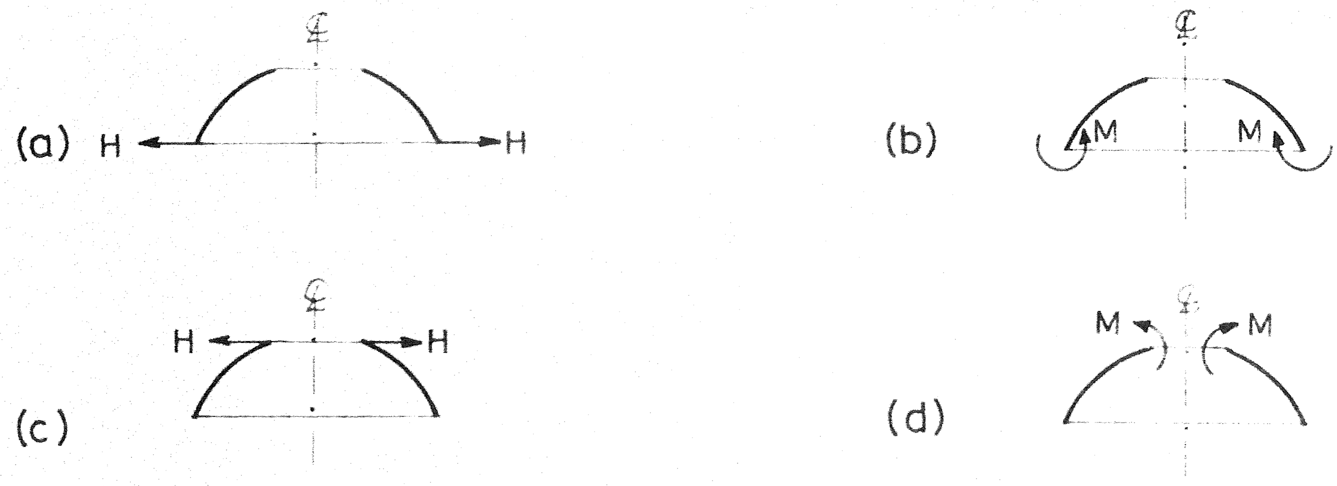


Fig. 3·12 Edge-loadings on spherical shell

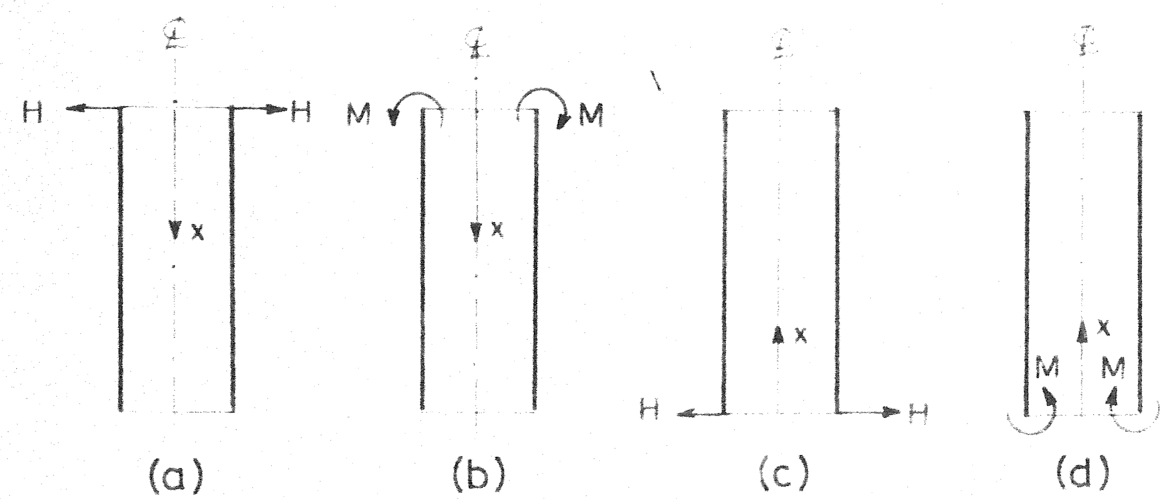


Fig. 3·13 Edge-loadings on cylindrical shell

Table 3.12 Bending solutions for open spherical shell - edge-loadings

Loading condition	Stress resultants and deformations	Algebraic expressions for stress resultants and deformations
1	2	3
	N_ϕ	$H\left[F_7(n) - \frac{F_4}{F_1} F_{10}(n) + \frac{F_2}{F_1} F_8(n)\right] \sin \phi_1 \cot \phi$
	N_θ	$- HK\left[- F_9(n) - 2 \frac{F_4}{F_1} F_7(n) + \frac{F_2}{F_1} F_{10}(n)\right] \sin \phi_1$
	Q_ϕ	$H\left[F_7(n) - \frac{F_4}{F_1} F_{10}(n) + \frac{F_2}{F_1} F_8(n)\right] \sin \phi_1$
	M_ϕ	$- \frac{HR}{2K}\left[- F_{10}(n) + 2 \frac{F_4}{F_1} F_8(n) - \frac{F_2}{F_1} F_9(n)\right] \sin \phi_1$
Refer Figures 3.12(a) and 3.6	M_θ	$\frac{HR}{2K} \left\{-\left[\frac{\cot \phi}{K} F_8(n) - \mu F_{10}(n)\right] + \frac{F_4}{F_1}\left[\frac{\cot \phi}{K} F_9(n) - 2 \mu F_8(n)\right] + \frac{F_2}{F_1}\left[\frac{\cot \phi}{K} F_7(n) + \mu F_9(n)\right]\right\} \sin \phi_1$
	δ	$- \frac{HRK}{Et}\left[- F_9(n) - 2 \frac{F_4}{F_1} F_7(n) + \frac{F_2}{F_1} F_{10}(n)\right] \sin \phi \sin \phi_1$

Contd...

1	2	3
	β	$- \frac{2HK^2}{Et} [- F_8(n) - \frac{F_4}{F_1} F_9(n) + \frac{F_2}{F_1} F_7(n)] \sin \phi_1$
	N_ϕ	$- \frac{2MK}{R} [\frac{F_6}{F_1} F_{15}(n) + \frac{F_5}{F_1} F_{16}(n) - \frac{F_3}{F_1} F_8(n)] \cot \phi$
	N_θ	$\frac{2MK^2}{R} [\frac{F_6}{F_1} F_{14}(n) + \frac{F_5}{F_1} F_{13}(n) - \frac{F_3}{F_1} F_{10}(n)]$
	Q_ϕ	$- \frac{2MK}{R} [\frac{F_6}{F_1} F_{15}(n) + \frac{F_5}{F_1} F_{16}(n) - \frac{F_3}{F_1} F_8(n)]$
	M_ϕ	$M [\frac{F_6}{F_1} F_{13}(n) - \frac{F_5}{F_1} F_{14}(n) + \frac{F_3}{F_1} F_9(n)]$
Refer Figures 3.12(b) and 3.6	M_θ	$- M \{ \frac{F_6}{F_1} [\frac{\cot \phi}{K} F_{16}(n) - \mu F_{13}(n)] - \frac{F_5}{F_1} [\frac{\cot \phi}{K} F_{15}(n) - \mu F_{14}(n$
		$- \frac{F_3}{F_1} [\frac{\cot \phi}{K} F_7(n) + \mu F_9(n)] \}$

Contd...

Table 3.12 (continued)

1	2	3
	δ	$\frac{2MK^2}{Et}\left[\frac{F_6}{F_1} F_{14}(n) + \frac{F_5}{F_1} F_{13}(n) - \frac{F_3}{F_1} F_{10}(n)\right] \sin \phi$
	β	$\frac{4MK^3}{EtR}\left[\frac{F_6}{F_1} F_{16}(n) - \frac{F_5}{F_1} F_{15}(n) - \frac{F_3}{F_1} F_7(n)\right]$
	N_ϕ	$H\left[\frac{F_9}{F_1} F_{10}(n) - 2 \frac{F_8}{F_1} F_8(n)\right] \cot \phi \sin \phi_2$
	N_θ	$2HK\left[- \frac{F_9}{F_1} F_7(n) + \frac{F_8}{F_1} F_{10}(n)\right] \sin \phi_2$
	Q_ϕ	$H\left[\frac{F_9}{F_1} F_{10}(n) - 2 \frac{F_8}{F_1} F_8(n)\right] \sin \phi_2$
	M_ϕ	$\frac{HR}{K}\left[\frac{F_9}{F_1} F_8(n) - \frac{F_8}{F_1} F_9(n)\right] \sin \phi_2$
Refer Figures 3.12(c) and 3.6	M_θ	$\frac{HR}{2K}\left\{\frac{F_9}{F_1}\left[- \frac{\cot \phi}{K} F_9(n) + 2\mu F_8(n)\right] - \frac{2F_8}{F_1}\left[\frac{\cot \phi}{K} F_7(n) + \mu F_9(n)\right]\right\} \sin \phi_2$

Contd...

Table 3.12 (continued)

1	2	3
	δ	$-\frac{2HRK}{Et}\left[\frac{F_9}{F_1} F_7(n) - \frac{F_8}{F_1} F_{10}(n)\right] \sin \phi_2 \sin \phi$
	β	$\frac{2HK^2}{Et}\left[\frac{F_9}{F_1} F_9(n) + 2\frac{F_8}{F_1} F_7(n)\right] \sin \phi_2$
	N_ϕ	$\frac{2MK}{R}\left[\frac{F_8}{F_1} F_{10}(n) - \frac{F_{10}}{F_1} F_8(n)\right] \cot \phi$
	N_θ	$\frac{2MK^2}{R}\left[-2\frac{F_8}{F_1} F_7(n) + \frac{F_{10}}{F_1} F_{10}(n)\right]$
	Q_ϕ	$\frac{2MK}{R}\left[\frac{F_8}{F_1} F_{10}(n) - \frac{F_{10}}{F_1} F_8(n)\right]$
	M_ϕ	$M\left[2\frac{F_8}{F_1} F_8(n) - \frac{F_{10}}{F_1} F_9(n)\right]$
Refer Figures 3.12(d) and 3.6	M_θ	$-M\left\{\frac{F_8}{F_1}\left[\frac{\cot \phi}{K} F_9(n) - 2\mu F_8(n)\right] + \frac{F_{10}}{F_1}\left[\frac{\cot \phi}{K} F_7(n) + \mu F_9(n)\right]\right\}$

Contd...

Table 3.12 (continued)

1	2	3
	δ	$\frac{2MK^2}{Et}\left[-2\frac{F_8}{F_1}F_7(n)+\frac{F_{10}}{F_1}F_{10}(n)\right]\sin\phi$
	β	$\frac{4MK^3}{EtR}\left[\frac{F_8}{F_1}F_9(n)+\frac{F_{10}}{F_1}F_7(n)\right]$

For K, F_i and $F_i(n)$ see Table 3.3 and Section 3.3.1.3.

Table 3.13 Bending solutions for long cylindrical shell - edge-loadings

Loading condition	Stress resultants and deformation	Algebraic expressions for stress resultants and deformations
1	2	3
	N_x	0
	N_θ	$2HRK_1 e^{-K_1\bar{n}} (\cos K_1 \bar{n})/L$
	M_x	$HLe^{-K_1\bar{n}} (\sin K_1 \bar{n})/K_1$
Refer Figures 3.13(a) and 3.7(a)	M_θ	μM_x
	Q_x	$(\sqrt{2}) He^{-K_1\bar{n}} \cos(K_1 \bar{n} + \pi/4)$
	δ	$HL^3 e^{-K_1\bar{n}} (\cos K_1 \bar{n})/(2DK_1^3)$
	β	$[HL^2 e^{-K_1\bar{n}} \sin(K_1 \bar{n} + \pi/4)]/[(\sqrt{2}) K_1^2 D]$
	N_x	0
	N_θ	$[(2\sqrt{2}) MRK_1^2 e^{-K_1\bar{n}} \cos(K_1 \bar{n} + \pi/4)]/L^2$

Contd...

Table 3.13 (continued)

1	2	3
	M_x	$(\sqrt{2})\, Me^{-K_1 \bar{n}} \sin(K_1 \bar{n} + \pi/4)$
Refer Figures 3.13(b) and 3.7(a)	M_θ	$\mu\pi M_x$
	Q_x	$-\ 2MK_1 e^{-K_1 \bar{n}} (\sin K_1 \bar{n})/L$
	δ	$[ML^3 e^{-K_1 \bar{n}} \cos(K_1 \bar{n} + \pi/4)]/[(\sqrt{2})\, DK_1^2]$
	β	$MLe^{-K_1 \bar{n}} (\cos K_1 \bar{n})/DK_1$
	N_x	0
	N_θ	$2HRK_1 e^{-K_1 \bar{n}} (\cos K_1 \bar{n})/L$
	M_x	$HLe^{-K_1 \bar{n}} (\sin K_1 \bar{n})/K_1$
Refer Figures 3.13(c) and 3.7(b)	M_θ	μM_x

Contd...

Table 3.13 (continued)

1	2	3
	Q_x	$-(\sqrt{2})\,He^{-K_1\bar{n}}\cos(K_1\bar{n}+\pi/4)$
	δ	$HL^3e^{-K_1\bar{n}}(\cos K_1\bar{n})/(2DK_1^3)$
	β	$-[HL^2e^{-K_1\bar{n}}\sin(K_1\bar{n}+\pi/4)]/[(\sqrt{2})\,K_1^2D]$
Refer Figures 3.13(d) and 3.7(b)	N_x	0
	N_θ	$-[(2\sqrt{2})MRK_1^2e^{-K_1\bar{n}}\cos(K_1\bar{n}+\pi/4)]/L^2$
	M_x	$-(\sqrt{2})\,Me^{-K_1\bar{n}}\sin(K_1\bar{n}+\pi/4)$
	M_θ	μM_x
	Q_x	$-2MK_1e^{-K_1\bar{n}}(\sin K_1\bar{n})/L$
	δ	$-[ML^3e^{-K_1\bar{n}}\cos(K_1\bar{n}+\pi/4)]/[(\sqrt{2})\,DK_1^2]$
	β	$MLe^{-K_1\bar{n}}(\cos K_1\bar{n})/DK_1$

For D and K_1 see Section 3.3.1.3.

Table 3.14 Membrane solutions for open conical shell – primary loadings

Loading condition	Stress resultants and deformations	Algebraic expressions for stress resultants and deformations
1	2	3
Dead weight Refer Figures 3.9(a) and 3.5	N_x	$-q\,x\,[(x_2/x)^2 - 1]/(2\sin\alpha)$
	N_θ	$q\,x\cos^2\alpha/\sin\alpha$
	δ	$q\,x^2[2\cos^2\alpha + \mu(x_2/x)^2 - \mu](\cot\alpha)/(2Et)$
	β	$q\,x[1 + 2\mu - 2(2+\mu)\cos^2\alpha - (x_2/x)^2](\cos\alpha)/(2Et\sin^2\alpha)$
Hydrostatic pressure loading. Refer Figures 3.9(b) and 3.5	N_x	$-\rho\,x\,\{(f/2\sin\alpha)[(x_2/x)^2 - 1] - (x/3)[(x_2/x)^3 - 1]\}\cos\alpha$
	N_θ	$\rho x(f - x\sin\alpha)\cot\alpha$
	δ	$\rho x^2\{(f/\sin\alpha) + (\mu f/2\sin\alpha)[(x_2/x)^2 - 1] - (\mu x/3)[(x_2/x)^3 - 1] - x\}(\cos^2\alpha)/(Et)$
	β	$\rho x\{(x/3)[8 + (x_2/x)^3] - (f/2\sin\alpha)[(x_2/x)^2 + 3]\}(\cos^2\alpha)/(Et\sin\alpha)$

Contd...

Table 3.14 (continued)

1	2	3
Equally distributed loading along the edge. Refer Figures 3.9(c) and 3.5	N_x	$-P x_2/(x \sin \alpha)$
	N_θ	0
	δ	$\mu P x_2 (\cot \alpha)/Et$
	β	$-P x_2 (\cos \alpha)/(Et \times \sin^2 \alpha)$

where q = the weight of shell per unit of its surface area,

ρ = unit weight of water,

p = equally distributed load per unit length of the edge.

Note:- 'f' for present case is equal to $x_2 \sin \alpha$.

Table 3.15 Membrane solutions for open spherical shell - primary loadings

Loading condition	Stress resultants and deformations	Algebraic expressions for stress resultants and deformations
Dead weight Refer Figures 3.10(a) and 3.6	N_ϕ	$-Rq(\cos\phi_2 - \cos\phi)/\sin^2\phi$
	N_θ	$-Rq[\cos\phi - (\cos\phi_2 - \cos\phi)/\sin^2\phi]$
	δ	$R^2q[-\cos\phi + (1+\mu)(\cos\phi_2 - \cos\phi)/\sin^2\phi](\sin\phi)/Et$
	β	$-Rq(2+\mu)(\sin\phi)/Et$

where q = the weight of shell per unit of its surface area.

Table 3.16 Membrane solutions for cylindrical shell - primary loadings

Loading condition	Stress resultants and deformations	Algebraic expressions for stress resultants and deformations
	N_x	0
Hydrostatic pressure loading. Refer Figure 3.10(b)	N_θ	$-\rho(L_1 - x)R$ for $x \leq L_1$
	δ	$-R^2\rho\ (L_1 - x)/Et$ for $x \leq L_1$
	β	$-R^2\rho/Et$ for $x \leq L_1$

The sign convention for deformations in general solutions also remain the same as given in Section 3.3.1.1.

For a shell of revolution with axisymmetric loading, the twisting moments, membrane inplane shears and the transverse shear in the circumferential direction are zero.

3.3.1.5 Applicability of membrane and bending solutions to reinforced concrete shells

The reinforced concrete shells, in general, are orthotropic in nature. The foregoing discussion is applicable to axisymmetrically loaded shells of revolution of homogeneous isotropic material. The membrane solutions given for finding the value of N_ϕ and N_θ can also be used directly for orthotropic shells because, the membrane is a statically determinate structure and stress resultants do not depend on the stiffness properties.

For special case of orthotropic shells in which the extensional and bending stiffnesses in meridional direction are equal to the respective stiffnesses in circumferential direction, the expressions described in the previous section can be used by merely replacing t and E by their modified values as described below:

In expressions for deformations obtained by membrane solution for primary loading, we set

$$Et = B(1 - \mu^2) \qquad (3.19$$

and, in expressions for stress resultants and deformations obtained by bending solutions for edge loading, we set

$$t = \sqrt{\frac{12D}{B}} \quad \text{and} \quad E = \frac{B(1-\mu^2)}{t} \tag{3.20}$$

where $B = B_\phi = B_\theta$ = extensional stiffness of the shell in meridional or circumferential direction.

$D = D_\phi = D_\theta$ = bending stiffness of the shell in meridional or circumferential direction.

The values of B and D can be found by the following expressions

$$B_i = \frac{E_i A_i}{1 - \mu^2} \quad \text{and} \quad D_i = \frac{E_i I_i}{1 - \mu^2}$$

where i refers to ϕ or θ,

A is the cross-sectional area of 1 m wide strip, and

I is the moment of inertia of 1 m wide strip.

It is further important to note here that in reinforced concrete water reservoirs the limit state of cracking governs the design. Therefore, they are designed such that tension in the concrete remains well within its cracking limit ensuring entire concrete section to be effective. It should also be noted that, for the loading combination considered for reservoir portion, the supporting shaft remains in compression only. Even for other loading combinations, a considerable length of the shaft near junction 2 will remain in axial compression throughout and the tension produced by wind induced ring moment and hoop tension will be well within the cracking limit.

As the section remains uncracked, the contribution to extensional and bending stiffnesses due to the area of reinforcement is small as compared to that contributed by the

area of concrete. Taking this into consideration, the stiffnesses of a shell element in the meridional and circumferential directions can be taken equal for all practical purposes even if the reinforcement in the two directions is slightly different. Moreover, these are not the absolute values of stiffnesses of any shell element which affect the result but are the relative values of stiffnesses of various shell elements interacting at the junctions. Thus, even if the contribution of reinforcement to stiffnesses is neglected, the solution so obtained will be quite accurate. In view of the foregoing discussion the stiffnesses in the two directions of a shell element turn out to be equal, thus facilitating the use of the solutions of homogeneous isotropic shells for reinforced concrete reservoirs taking E as the Youngs modulus of elasticity of concrete and t as the thickness of the shell. Results so obtained are reasonably accurate, in particular the hoop tensions at locations which usually govern the design of conical tank wall remain unaffected.

3.3.2 Elastic analysis for finding the design forces in the supporting shaft

The analysis of the supporting structure is carried out by considering the shaft as a cantilever fixed at the base. The various loading combinations to be considered for finding the design forces in the shaft are those given in Table 3.2. The analysis for the case of dead load + weight of water under tank full condition has been already discussed in the previous section. In order to carry out the analysis

for the other two load combinations, the design values of wind pressures and the seismic forces are required to be found out. The design wind pressure is obtained by following the procedure given in Section 3.2.1 and using an appropriate value of partial safety factor given in Table 3.2.

For finding the design seismic forces, the response spectrum method is used to find the value of horizontal seismic coefficient. IS: 1893 specifies a simplified method. In this method, elevated tanks are considered to be the systems with a single degree of freedom with their mass concentrated at their centres of gravity. The period of free vibration T, in seconds, is given by

$$T = 2\pi\sqrt{\frac{\Delta}{g}} \tag{3.21}$$

where Δ = the static horizontal deflection at the top of the tank under a static horizontal force equal to a weight W acting at centre of gravity of tank; and

g = acceleration due to gravity.

The value of the weight W, to be used in calculating the deflection Δ, is taken as equal to the dead load of the tank and one-third the weight of the supporting structure under empty tank condition. When tank is full, the weight of contents is to be added to the weight under empty conditic

Using the period T as calculated by Eq. (3.21) and damping as 5 percent of the critical value for the concrete structure, the value of average acceleration coefficient $\frac{S_a}{g}$

can be read from average acceleration spectra given in Figure 3.1.

In order to find out the value of horizontal seismic coefficient α_h using Eq. (3.2), suitable values for β and I are to be adopted. For guidance IS: 1893-1975 has specified the values for these coefficients. In the present study the value of I is taken as 1.5 and the value of β as 1.0 as it covers most of the types of foundation under all soil conditions.

The lateral force is taken **as equal** to:

$$\alpha_h W$$

where α_h = value of horizontal seismic coefficient as obtained by Eq. (3.2), and

W = weight as defined earlier.

This force is multiplied by appropriate partial safety factor to obtain the design lateral force. The design lateral force shall be assumed to act at the centre of gravit of the tank horizontally in the plane in which the structure is assumed to oscillate. This is for the purposes of carrying out the analysis of the supporting structure.

Once the design values of wind and seismic forces are obtained, the axial force and the bending moment at any horizontal section of the support can easily be obtained. The most stressed section is the one at the top of the foundation.

In longitudinal cross-sections of shell-type supports horizontal loads, particularly wind loads, induce

bending moments, normal, and transverse forces. In cylindrical supports they may be found as a first approximation by analysing elastic rings 1 m high visualized in the support between two horizontal cross-sections and equilibrated by the horizontal load acting on the tower at the level where the considered ring is located and by the internal tangential forces that affect the ring cross-sections. The approximate value of the induced ring moment, when the wind load is distributed over the circumference, is equal to:

$$0.33\ W_p\ r^2 \text{ in kNm/m}$$

where W_p = design value of wind pressure at any level in kN/m^2, and

r = mean radius of the shaft in m.

The hoop force and the shear force come out to be of negligible value.

3.4 Design Considerations

3.4.1 Materials

The recommended grades of concrete for reinforced concrete water tanks, according to IS: 3370 (Part I), are M20, M25, etc. but a mix weaker than M20 is not used. In view of the early thermal cracking problem associated with richer mixes, M20 grade is most commonly used and is preferred in the current study also. The minimum quantity of cement is kept not less than 3.3 kN/m^3 in the concrete mix. In order to make dense impervious concrete, well graded

aggregate with adequate water-cement ratio and means of thorough compaction should be used. It should be recognized that common and more serious causes of leakage, other than cracking, are defects such as segregation and honeycombing and in particular all joints are potential source of leakage. Special measures should be taken to avoid all these defects. Traditionally mild steel reinforcement is used for water tanks because of the low allowable stresses permitted in the design to avoid cracks. But the superior bond properties of deformed bars, allowing for shorter laps and providing better crack distribution, will compensate for their slight extra cost even if their higher strength cannot be used to the fullest extent in the reservoir portion. However, the use of deformed bars will result in substantial saving in reinforcement required for the supporting shaft. Therefore, high yield strength deformed bars of 415 grade having characteristic strength of 415 MPa have been adopted for the present work.

3.4.2 Cracking phenomenon and its control

In liquid retaining structures, serviceability limit state of cracking is the one which, in most cases, governs the design of the reservoir portion and requires due consideration for crack control in the design procedure. The complex and semi-random phenomenon of cracking has been tackled through experimental work. Base et al. (1966), Beeby (1971, 1979, 1983), Broms (1964, 1965), Broms and Lutz (1965), Desayi (1976a, 1976b, 1980), Nawy (1970, 1972) and Nawy and Blair (1971) are among the investigators who have

contributed significantly to the present state-of-art. Although many expressions for estimating the flexural crack width and crack spacing have been proposed, the significance of some of the basic variables is still being debated. Therefore, the procedure for crack control should be used only as a guide line rather than as a rigid rule. Formulae for flexural surface crack width are available in some of the latest codes. However, an equation which can be used with the confidence required for the design of water retaining structures, particularly when subjected to direct tension, is not yet available. Therefore, safety in respect of cracking due to direct tension, in water tanks, is ensured only by designing the crack resistant structure by limiting the direct and bending tensile stresses both in concrete and steel to a certain level. To deal with such problems, BS 5337 specifies an alternative method of design in its clauses 6, 9 and 12. A similar design procedure, given in IS: 3370 (Part II) is adopted in the current study with a little modification in view of some latest provision in BS 5337. This procedure adopts the following measures to ensure watertightness:

(i) Thickness of the tank wall is chosen such that the direct tensile stress as well as flexural tensile stress in concrete, computed on the equivalent uncracked section basis, does not exceed the permissible values 1.2 MPa and 1.7 MPa respectively for M20 concrete.

(ii) The hoop reinforcement is designed to resist the complete hoop tension by restricting the permissible

stress in deformed bars to 100 MPa in accordance with 'deemed to satisfy' provision of BS 5337 for almost no crack condition.

(iii) The reinforcement required to resist bending moment is based on the cracked section with the same value of permissible stress as given above in (ii).

Although supporting shaft is subjected to combined axial load and bending moment and there is hardly any chance of crack width being critical, it may sometimes require crack width calculations for dead weight + wind/seismic load under empty tank condition. IS: 456 has given a guide line that the width of surface flexural cracks, in general, should not exceed 0.3 mm for mild exposure. CP 110 has given the following equation for estimating the width of surface flexural cracks which is used in the present work:

$$w_{cr} = \frac{3a_{cr}\,\varepsilon_m}{1 + 2\,\dfrac{(a_{cr} - c_{min})}{(h - d_n)}} , \tag{3.22}$$

ε_m is the average strain in the member at the point of surface under consideration and is given by equation:

$$\varepsilon_m = \frac{\varepsilon_s y}{d - d_n} - \frac{1.2\, b_t h y}{A_s (h - d_n)\, f_y} \times 10^{-3} , \tag{3.23}$$

where a_{cr} = distance from the point under consideration to the surface of the nearest longitudinal bar,

c_{min} = minimum cover to the tension reinforcement,

h = depth of the member,

d_n = depth of neutral axis,

ε_s = strain in steel at service loads,

y = distance from neutral axis to level of point considered,

d = effective depth of the member,

b_t = width of the member at centroid of tension reinforcement, and

f_y = characteristic strength of reinforcement.

The second term in Eq. (3.23) represents the stiffening effect of concrete in the tension zone. The allowable value of surface flexural crack width, for shaft, is taken as 0.3 mm in the present study.

3.4.3 Design of top dome, inner cylinder and ring beams

It is found from actual analysis that the membrane and the bending forces in the top spherical dome and the inner cylinder are of such order that they do not demand thick sections. However, practical considerations preclude consideration of any thickness less than 75 mm for the top dome and 125 mm for the inner cylinder. The area of reinforcement is also governed by the minimum value of 0.25% except near the edges, in some cases. The edge forces die out fast and extra reinforcement required at edges, if any, can be curtailed within a short distance. For economy, the rise of dome should be in the range of 0.25 to 0.4 times its radius and in order to avoid necessity of formwork for the top surface, the semi-central angle should not exceed 40°. Based

on these considerations the rise of the dome is kept as one-third of its radius. Cost of the dome and the inner cylinder in the objective function is computed, for a 75 mm thick dome with rise equal to one-third of its radius and 125 mm thick cylinder and the reinforcement of 0.25% in both directions for these two elements except for some extra reinforcement in cylindrical portion for a short length near junction 2, in some cases.

Although it is possible to provide the required horizontal reactions at junctions without providing ring beams, such an arrangement results in high tension in the edge regions and often requires heavy reinforcement and increased thicknesses of the elements meeting at these junctions. Moreover, in liquid retaining structures, it is not desirable to have regions of high hoop tension, particularly at junctions, as they may give rise to wide cracks. Therefore, ring beams provided at junctions 1 and 2 (see Figure 3.3) eliminate all such undesirable effects and can be easily reinforced. Dimensions of ring beams can be chosen as design variables, but cost of ring beams in relation to the cost of the system is very small. Besides efficiency of any optimization technique will decrease with the increase in the number of design variables. As such, both the ring beams are chosen to be square in section with 1.5 t_c as the side, t_c denoting the thickness of the conical reservoir. Reinforcement required for ring beams is obtained from the hoop tension consideration.

3.4.4 Distribution and arrangement of reinforcement in various elements

Usually distribution and arrangement of reinforcement depend upon the forces in the structure at service loads. As discussed in the preceding section the top dome and the inner cylindrical portion will have a uniform distribution of reinforcement in both the directions except near the edges where some extra reinforcement may be required, in a few cases.

In order to control the cracks in the conical reservoir portion at service loads more hoop reinforcement should be placed at locations where hoop tensions are larger as obtained from the elastic analysis. The Figure 3.14 shows the actual general pattern of variations in the values of stress resultants along the wall of the conical tank in the x-direction for reservoirs of different capacities. It can be seen that the hoop tension is larger in about two-thirdslength of the lower portion except for a very short length near the narrow end of the cone. Thus, the curtailment in hoop reinforcement for a short length, that too, at the narrow end of the cone will not be of much use. Whereas, in about one-third length of the upper portion of the cone, hoop tension decreases rapidly. Therefore, curtailment of reinforcement in this portion is meaningful and this results in considerable saving. The common practice of curtailing the reinforcement where forces are smaller, leads to the stepped distribution. Although the geometric parameters fixing the positions of points of curtailment can be taken as design variables, but this will increase the size of optimization problem considerably

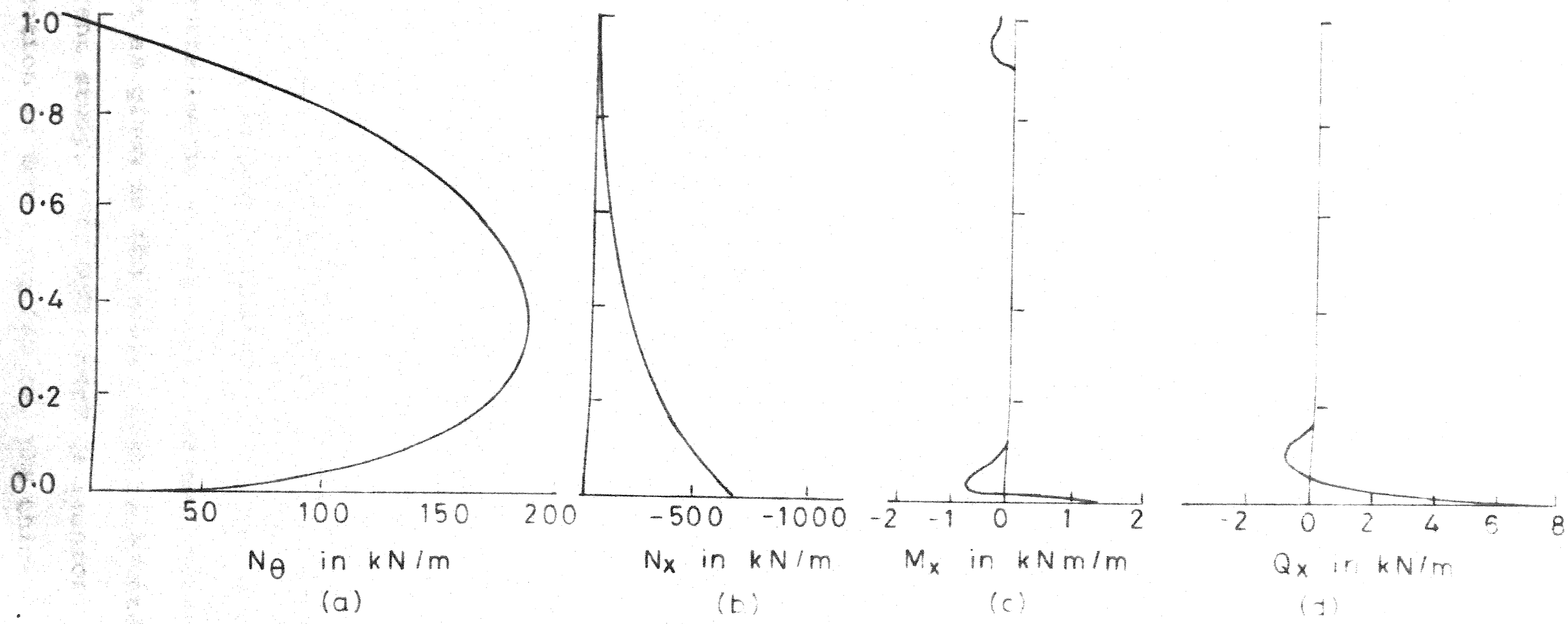


Fig. 3·14 General pattern of variation of stress resultants along the wall of the conical tank

(A typical case of a tank with capacity 300 m^3, R_2=1·125 m, R_1/R_2=5·0)

without much gain than those fixed by intuitive trial and error method. It is important to note here that hoop tension variation for different tank capacities is found to be very much similar to that given in the Figure 3.14(a) and for this variation the distribution of hoop reinforcement given in the Figure 3.15, which also indicates the points of curtailment, is found to be the economical one for most of the cases and hence is adopted throughout the present study. The percentage of hoop reinforcement in the different regions and the thickness of the cone are taken as design variables in the optimization process. The membrane and bending forces, in the conical portion in x-direction do not demand the reinforcement more than the minimum of 0.25% except near the edges, in some cases. Hence, in x-direction, the conical portion will have a uniformly distributed reinforcement throughout except at the edges where some extra reinforcement may have to be provided in some cases.

For the supporting shaft, the area of reinforcement required in vertical direction will depend upon the bending moment and the axial force acting on any horizontal section of the shaft. These forces will be maximum at the section just above the foundation and minimum at the top of the shaft. A stepped distribution of reinforcement with points of curtailments at one-third and two-thirds heights of the shaft as given in the Figure 3.16 is adopted throughout the current study. The percentage of reinforcement in vertical direction in three regions and the thickness of the shaft

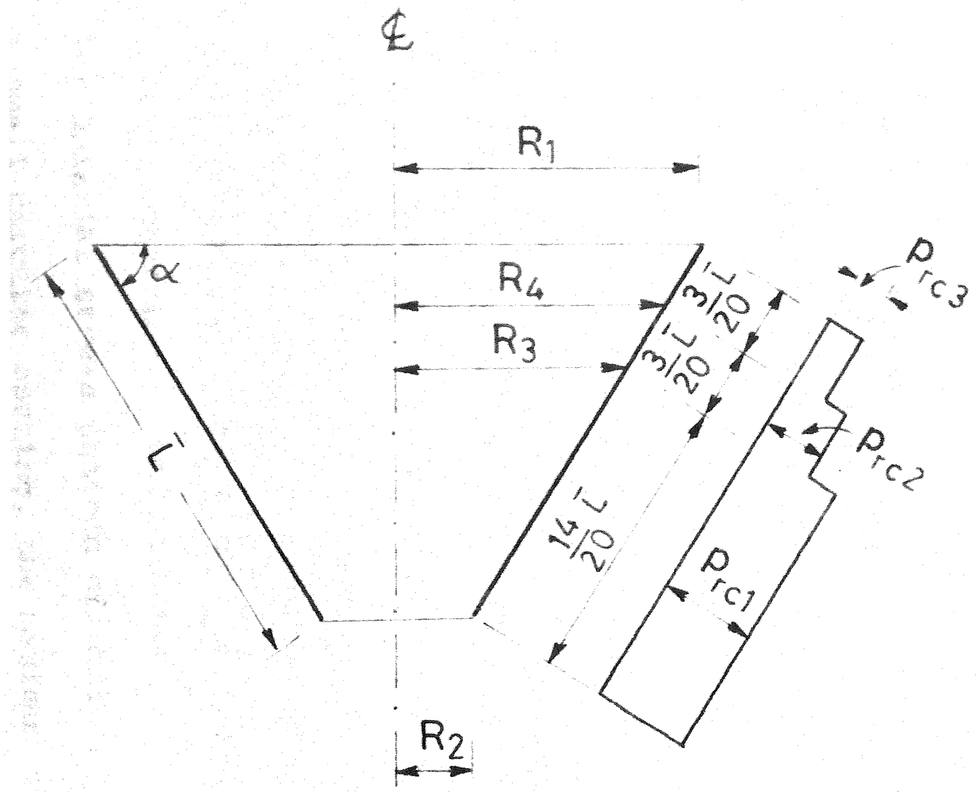


Fig. 3·15 Distribution of hoop reinforcement for the conical tank

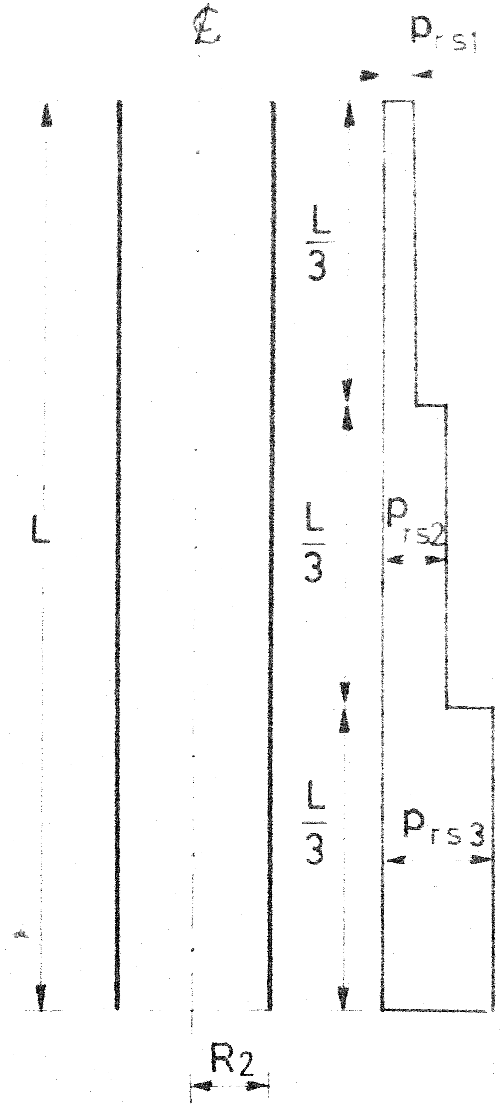


Fig. 3·16 Distribution of vertica[l] reinforcement for the supporting shaft

are taken as design variables. A minimum reinforcement of 0.25% is found to be adequate in circumferential direction.

In conical, inner cylindrical and supporting cylindrical shells reinforcements in two directions are placed on both the sides, 50% on each face, in a staggered fashion. This is to take care of the sudden jumps in the bending moment (from positive to negative) near the junction 2. Any imperfection due to construction by way of hump or necking will be taken care of by providing reinforcement on both the faces. The cover provided for conical and the inner cylindrical shells is 40 mm and for the support it is kept equal to 30 mm. The hoop reinforcement is placed on the inner side of the meridional reinforcement.

3.4.5 Openings

3.4.5.1 Stress concentration

Small and big openings are required to be provided in water towers for different purposes. These openings result in weakening of the structural element locally at the points of their location and in stress concentration around the edges of the opening. The latter is more serious for small openings and the value of stress concentration factor (SCF) depends upon the shape of the opening and the general stress pattern occurring in the structural element at that location. For uniaxial stress pattern of uniform intensity, in case of a small circular opening, the maximum stress is as high as three times the value of the average stress and it occurs at

the ends of the diameter of the hole which is at right angles to the uniaxial stress direction. Whereas, at the ends of the diameter parallel to the uniaxial stress direction, a stress equal to the average stress but of opposite nature occurs. These stresses damp out rapidly within a distance equal to the radius of the opening and become equal to the average stress at a distance five times the radius of the opening as shown in the Figure 3.17. For biaxial stress pattern the stress concentration factor can be obtained by superposing the foregoing results of the uniaxial case. In large openings such as the one for the door provided at the bottom of the supporting shaft for access to the reservoir portion, the stress concentration factor at mid points of vertical edges of the opening can be obtained by approximating the opening as an elliptical one for which SCF = $1 + 2\frac{b}{a}$, where 'b' is the width of the opening and 'a' is its height. For an average size of door opening the value of SCF will be between 1.5 to 2 for uniaxial stress pattern of uniform intensity acting parallel to the axis of the shaft. Whereas, the stress at the mid points of top and bottom edges of the opening will be equal to the intensity of uniaxial stress but of opposite nature.

3.4.5.2 Reinforcing of openings

A sufficient material should be added to compensate the weakening effect of the opening as well as to take care of stress concentration around the edges of the opening. The reinforcing material be placed immediately adjacent to the

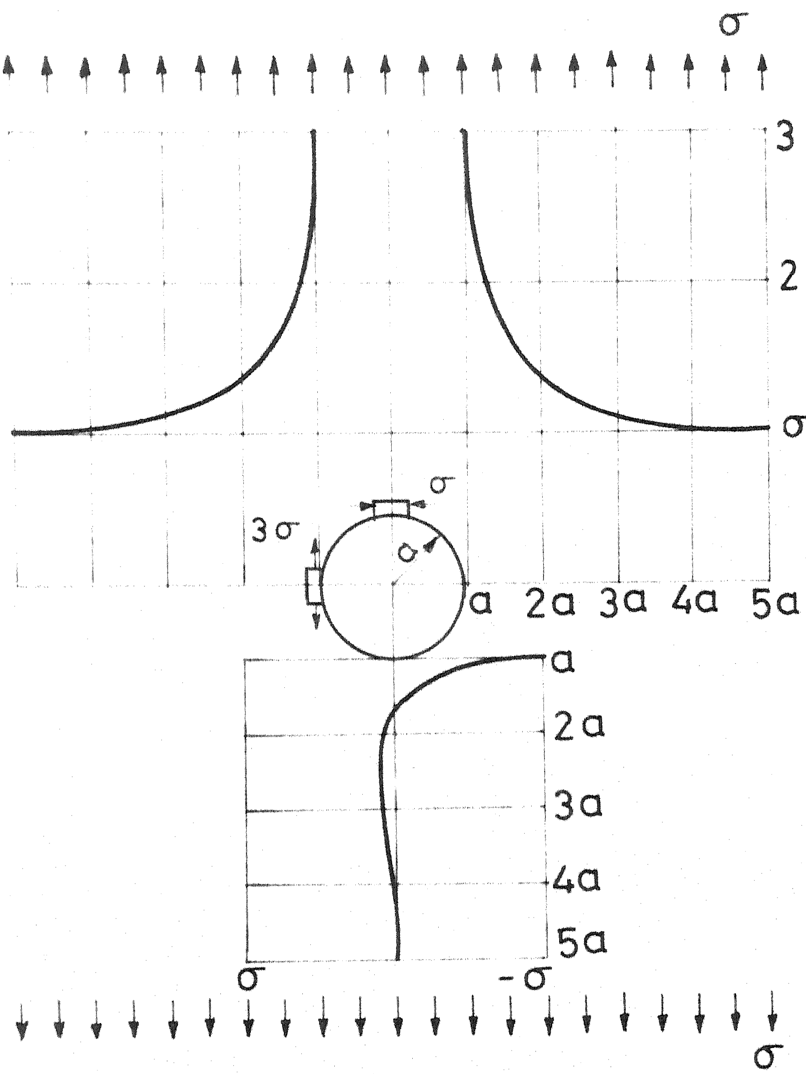


Fig. 3·17 Variation in stress in a shell element containing a circular hole and subjected to uniform stress

opening but suitably disposed in a profile and contour so as not to introduce an over-riding stress concentration itself. Reinforcement of an opening cannot be unquestionably obtained by adding huge amounts of material because this has the reverse effect and creates a 'hard spot' on the shell which does not allow natural growth under general strain pattern occurring throughout the shell element. This results in local overstressing causing the failure of the structure which is similar to the pinching of a baloon. Usually, in reinforced concrete structures, best results are obtained by gradual increase in thickness of the concrete section and in reinforcing steel. In case of door opening in the vertical supporting shaft of a water tower the horizontal edges should be thickened with gradual increase in thickness so as to provide lintel action to take care of vertical forces at each of the edges. These edges are required to take a small amount of vertical load as most of it will be transferred through the vertical reinforcement which will be bent to pass by the sides of the vertical edges of the opening. The amount of the reinforcement along the horizontal edges should be increased to take care of the tensile stress produced due to stress concentration effects. The vertical edges of the opening are stiffened by thickening the edges with gradual increase in thickness so that the values of effective area of cross section and moment of inertia become at least equal to the respective values of a horizontal cross section of the shaft without opening. As far as possible, sharp corners should be avoided and the

opening can be made approximately elliptical in shape so as to avoid stress concentration at corners as well as to facilitate the gradual bending of vertical bars to pass by the sides of the opening.

3.5 Limit Analysis

Although, the limit state of collapse, in this particular case, will not usually govern the design, but in order to find out the margin of safety against collapse, the limit analysis is carried out.

3.5.1 Applicability of limit analysis

The general theory of limit analysis of rigid-plastic structures has been rigorously formulated by Prager (1952). The theory of limit analysis can as well be applied to elastic-plastic or brittle-plastic materials. This would be possible, if it is first established experimentally that such structures deform considerably at practically constant load intensity before collapse. It should also be possible to formulate the conditions of failure. Under-reinforced structures fall under this category. In the case of reinforced concrete shells, because the percentage of reinforcement is small, plastic behaviour is fairly well established. Experimental investigations, by Baker (1953), Ernest and Marlette (1954), Sawczuk (1961) and Das (1983), strongly indicated that collapse mechanisms and limit loads do exist for reinforced concrete shells. Adidam and Subramanyam (1982) applied limit analysis to obtain collapse water pressure for reinforced

concrete cylindrical tanks. They used lower bound approach adopting the failure criterion based on the work of Sawczuk and Olszak (1961), whereas Adidam and Kalwar (1984) obtained the collapse water pressure through upper bound approach for a case of reinforced concrete conical water tank. This method is used in the current study also for finding the collapse water pressure of the conical tank.

Water tower considered in this chapter is mainly composed of four elements, viz., top spherical dome, conical tank, inner cylindrical shell and the supporting shaft. Although, the limit analysis for each element can be carried out, the possibility of collapse of the top dome or the inner cylindrical portion prior to the collapse of the conical tank or the supporting shaft is negligible. This is because of the fact that both top dome and the inner cylindrical portion are primarily subjected to membrane compression and will collapse at a higher load factor than for the other two elements. Moreover, the possibility of combined mechanism is also rare. Therefore, in the present study, limit analysis of conical tank for axisymmetric case of loading that is dead load + water pressure under tank full condition is carried out. The supporting shaft can easily be analysed as a cantilever subjected to combined axial and lateral forces.

3.5.2 Limit analysis of conical tank

As already mentioned, the limit analysis of the conical tank is carried out as per upper bound approach. The water pressure at collapse is obtained by considering all the

possible collapse mechanisms of the tank. The design parameters of the tank are fixed up by the serviceability requirements. The Figure 3.18 shows the details of the conical tank with hydrostatic pressure loading at collapse and the ultimate hoop tension capacity distribution based on serviceability limit state of cracking (see Section 3.4.4). The relative values of ultimate hoop tension capacities N_{t1}, N_{t2}, N_{t3} and l_1 and l_2 will depend on the shell parameters.

3.5.2.1 Collapse mechanism

In this case, the conical reservoir is provided with a roof and a supporting structure, the ring beams and other elements meeting at junctions usually will not allow the conical portion to expand or contract considerably at the junctions and the only possible collapse mechanism will be of the form as shown in the Figure 3.19. The hinge circle 1 will be always just near the bottom junction and obviously remains in the region (N_{t1}). The hinge 2 will also be in region (N_{t1}), but hinge circle 3 may be formed in one of the three regions depending upon the shell parameters. Based upon the position of hinge circle 3, three different expressions for internal work will be obtained. However, the expression for external work, due to hydrostatic pressure and self weight of the shell, will essentially remain same for all the cases.

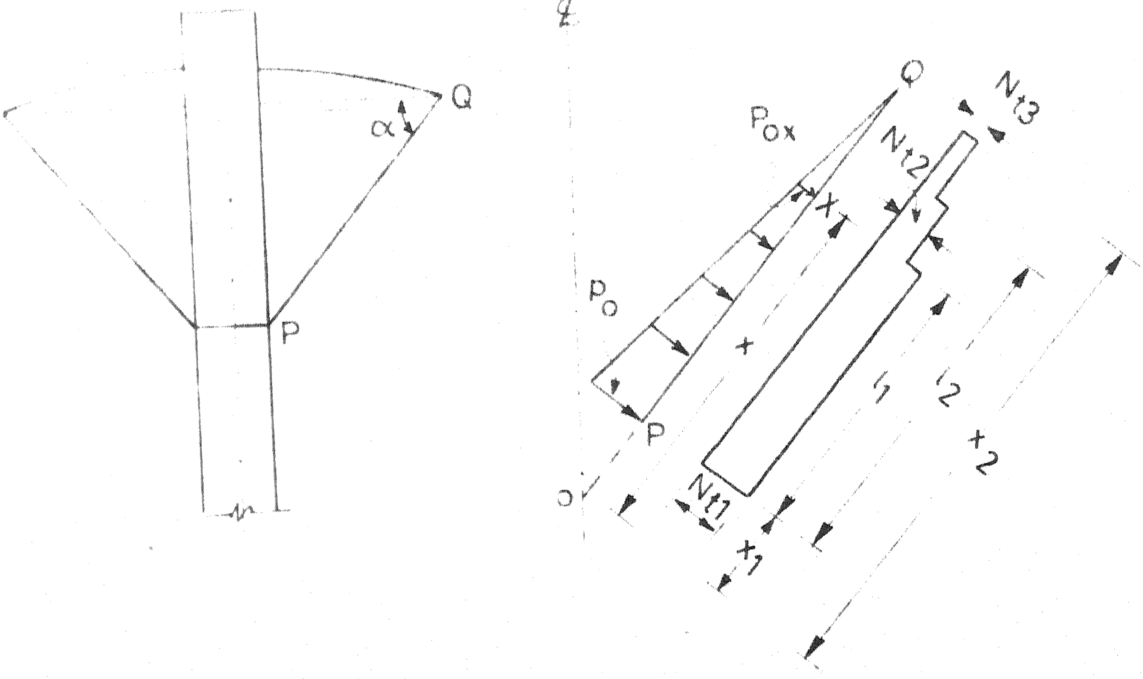


Fig. 3·18 Details of conical tank

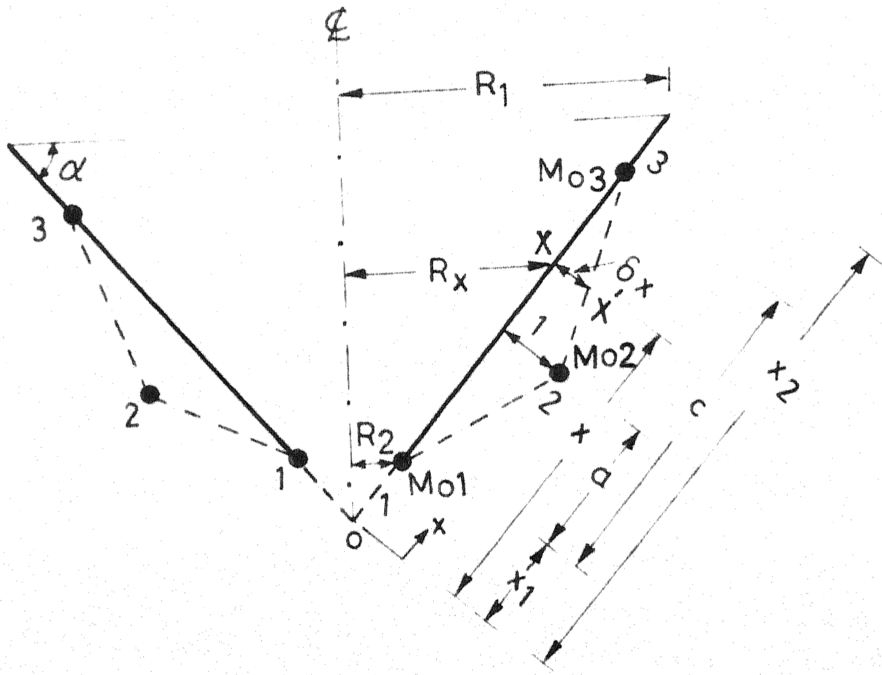


Fig. 3·19 The form of the possible collapse mechanism and its details

3.5.2.2 Internal work

The total internal work is comprised of the energy absorbed at plastic hinge circles and in the meridional yield lines, energy dissipated by extension of the circumferential reinforcement. In a conical reservoir the value of meridional thrust 'N_x' is considerable and in some cases it may reduce the ultimate moment of resistance of the shell section. In the Figure 3.19, M_{01}, M_{02} and M_{03} are the appropriate ultimate moments of resistance at the hinge circles 1, 2 and 3 respectively. The expression for the internal energy absorbed at plastic hinge circles can be written as under

$$I_{w1} = 2\pi \left[\frac{x_1}{a} M_{01} + \frac{c(x_1 + a)}{a(c - a)} M_{02} + \frac{(x_1 + c)}{(c - a)} M_{03}\right] \cos\alpha \qquad (3.24)$$

The internal work by extension of circumferential reinforcement will depend upon the position of hinge circle 3. In the Figure 3.19, O is the origin, distances x_1 and x_2 define the position of bottom and top of the conical shell, δ_x is the displacement of any point X on the shell surface, R_1, R_2 and R_x are the radii at top, bottom and point X of the conical shell. The point X will occupy the new position X' after virtual displacement and thus the increase in the length of the radius at X can be given for the two regions as

$$\delta_{Rx} = \delta_x \sin\alpha = \frac{x - x_1}{a} \sin\alpha \qquad \text{for} \quad x_1 \le x \le x_1 + a \qquad (3.25)$$

and

$$\delta_{Rx} = \delta_x \sin\alpha = \frac{x_1 + c - x}{c - a} \sin\alpha$$

$$\text{for} \quad x_1 + a \leq x \leq x_1 + c \qquad (3.26)$$

Thus the corresponding increase in circumferential length at this point can be given as

$$\delta l = \frac{2\pi(x - x_1)}{a} \sin\alpha \qquad \text{for} \quad x_1 \leq x \leq x_1 + a \qquad (3.27)$$

and

$$\delta l = \frac{2\pi(x_1 + c - x)}{c - a} \sin\alpha \qquad \text{for} \quad x_1 + a \leq x \leq x_1 + c \qquad (3.28)$$

Using the appropriate value for δl given by Eqs. (3.27) and (3.28), the expressions for internal work due to extension of circumferential reinforcement for the three cases, depending upon the position of hinge circle 3 (Figure 3.20), can be written as

for $l_2 \leq c \leq x_2$

$$I_{w2} = \int_{x_1}^{x_1+a} 2\pi N_{t1} \frac{(x - x_1)}{a} \sin\alpha \, dx$$

$$+ \int_{x_1+a}^{x_1+l_1} 2\pi N_{t1} \frac{(x_1 + c - a)}{(c - a)} \sin\alpha \, dx$$

$$+ \int_{x_1+l_1}^{x_1+l_2} 2\pi N_{t2} \frac{(x_1 + c - a)}{(c - a)} \sin\alpha \, dx$$

$$+ \int_{x_1+l_2}^{x_1+c} 2\pi N_{t3} \frac{(x_1 + c - a)}{(c - a)} \sin\alpha \, dx \qquad (3.29)$$

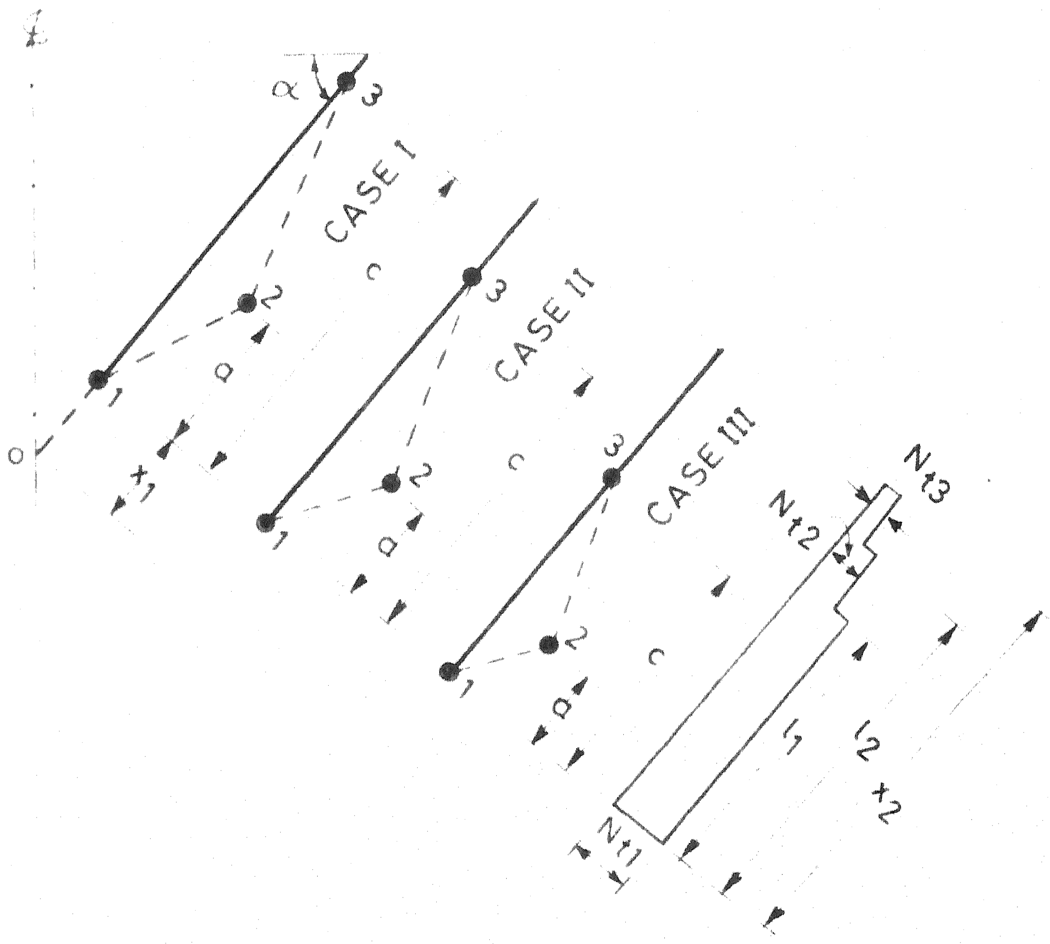


Fig. 3·20 Details of collapse mechanisms

for $l_1 \leq c \leq l_2$

$$I_{w2} = \int_{x_1}^{x_1+a} 2\pi N_{t1} \frac{(x - x_1)}{a} \sin\alpha \, dx$$

$$+ \int_{x_1+a}^{x_1+l_1} 2\pi N_{t1} \frac{(x_1 + c - a)}{(c - a)} \sin\alpha \, dx$$

$$+ \int_{x_1+l_1}^{x_1+c} 2\pi N_{t2} \frac{(x_1 + c - a)}{(c - a)} \sin\alpha \, dx \tag{3.30}$$

for $a < c \leq l_1$

$$I_{w2} = \int_{x_1}^{x_1+a} 2\pi N_{t1} \frac{(x - x_1)}{a} \sin\alpha \, dx$$

$$+ \int_{x_1+a}^{x_1+c} 2\pi N_{t1} \frac{(x_1 + c - a)}{(c - a)} \sin\alpha \, dx \tag{3.31}$$

On simplification, the Eqs. (3.29) through (3.31) will reduce to the following three equations.

Case I ($l_2 \leq c \leq x_2$)

$$I_{w2} = K_2 \, a \, N_{t1} + \frac{K_2}{c - a} \left[(l_1 - a)(2c - l_1 - a)N_{t1} + (l_2 - l_1)(2c - l_2 - l_1)N_{t2} + (c - l_2)^2 N_{t3}\right] \tag{3.32}$$

Case II ($l_1 \leq c \leq l_2$)

$$I_{w2} = K_2 \, a \, N_{t1} + \frac{K_2}{c - a} \left[(l_1 - a)(2c - l_1 - a)N_{t1} + (c - l_1)^2 N_{t2}\right] \tag{3.33}$$

Case III ($a < c \leq l_1$)

$$I_{w2} = K_2 \, c \, N_{t1} \tag{3.34}$$

where $K_2 = \pi \sin \alpha$

3.5.2.3 External work

In the Figure 3.18, p_o is the maximum hydrostatic pressure intensity at collapse. The pressure intensity p_{ox} at any point X can be obtained as

$$p_{ox} = \frac{x_2 - x}{x_2 - x_1} \, p_o$$

where $p_o = \gamma_L (x_2 - x_1) \sin \alpha$,

ρ = unit weight of water, and

γ_L = partial safety factor for loads.

The expression for the work done by hydrostatic pressure through virtual displacement (Figures 3.18 and 3.19) can be obtained as follows

$$E_{w1} = \frac{2\pi p_o \cos \alpha}{a(x_2 - x_1)} \left[\int_{x_1}^{x_1+a} x(x_2 - x)(x - x_1) \, dx \right]$$

$$+ \frac{2\pi p_o \cos \alpha}{(c - a)(x_2 - x_1)} \left[\int_{x_1+a}^{x_1+c} x(x_2 - x)(x_1 + c - x) \, dx \right] \tag{3.35}$$

On simplification of Eq. (3.35) the following expression is obtained

$$E_{w1} = K_3 \{ \frac{(x_2 + x_1)(a^2 + 3x_1^2 + 3x_1 \, a)}{3}$$

$$- \frac{a^3 + 4a^2x_1 + 6ax_1^2 + 4x_1^3}{4} - \frac{x_2x_1(a + 2x_1)}{2}$$

$$+ \frac{(x_2x_1 + cx_2)(c + a + 2x_1)}{2} - \frac{x_2 + x_1 + c}{3} \cdot$$

$$[c^2 + a^2 + ca + 3x_1(c + a + x_1)] + \frac{1}{4} [(c + a)(c^2 + a^2)$$

$$+ 4x_1(c^2 + a^2 + ca) + 6x_1^2(c + a) + 4x_1^3]\} \qquad (3.36)$$

where $K_3 = \dfrac{2\pi p_o \cos\alpha}{x_2 - x_1}$

If ρ_c is the unit weight of concrete, then the dead load at collapse w_o per unit surface area of the conical shell can be written as under

$$w_o = \gamma_L \rho_c t_c$$

where t_c = thickness of the conical shell.

Proceeding in a similar manner, external work due to self weight of the shell can be obtained. The final expression comes out as follows:

$$E_{w2} = K_4 [\frac{a^2 + 3x_1^2 + 3x_1a}{3} - \frac{x_1(a + 2x_1)}{3}$$

$$+ \frac{(x_1 + c)(c + a + 2x_1)}{2} - \frac{c^2 + a^2 + ca}{3}$$

$$- x_1(c + a + x_1)] \qquad (3.37)$$

where $K_4 = 2\pi w_o \cos^2\alpha$.

Thus, the principle of virtual work gives

$$I_{w1} + I_{w2} = E_{w1} + E_{w2} \qquad (3.38)$$

The Eq. (3.38) can be solved, for all the possible combinations of a and c, to obtain the value of p_o for each such combination. The lowest value of p_o so obtained will correspond to the hydrostatic pressure at collapse. In the present study one hundred and twenty one such mechanisms are tried to find out the collapse pressure p_o. In Eq. (3.38), the value of E_{w2} (the external work due to the self weight of the shell) is found to be negligible as compared to that of E_{w1}. The contribution of E_{w2} to total external work is hardly 5%. Similarly the contribution of I_{w1} (the internal energy absorbed at plastic hinge circles) found to be about 10% of the total internal energy. Hence, for simplification, values of E_{w2} and I_{w1} may be neglected in finding the collapse pressure p_o.

3.5.3 Limit analysis of cylindrical supporting shaft

In case of small capacity tanks, the design of the shaft is found to be governed by serviceability limit state of deflection. Whereas in some cases of tanks having capacities 500 kl to 1000 kl, ultimate limit state of strength of a local section of the cylindrical shaft when considering an element of unit width, may be critical. However, the limit state of collapse of the shaft is never found to govern the design. In order to estimate the margin of safety against collapse, the limit analysis is carried out by idealising the shaft as a cantilever of hollow circular section. It is of interest to note here that under combined axial and lateral forces, a local section of shell element of unit width will have

almost uniform axial strain across the thickness of the shell. Therefore, to safeguard against failure of such local sections, the maximum strain in the extreme fibre of the hollow circular section should not exceed 0.002 as per IS: 456.

3.5.4 Material properties and assumptions

In order to find out the ultimate strength of a section under axial, bending or combined axial and bending forces, the stress-strain relations used, for both concrete and steel, are as given in IS: 456. For obtaining the design strengths of these materials the partial safety factors specified by the Code are 1.5 and 1.15 for concrete and steel respectively.

The following assumptions, as per IS: 456, have been employed in the limit analysis:

(a) Plane sections normal to the plane of bending remain plane after bending.

(b) The strain in concrete at the outermost compression fibre in flexure reaches a value of 0.0035 at failure.

(c) The distribution of compressive stress in concrete is defined by an idealised stress-strain curve as shown in the Figure 3.21(b).

(d) The tensile strength of concrete is ignored.

(e) The stress in the reinforcement is derived from the representative stress-strain diagram of the steel, which is shown in the Figure 3.21(a).

(f) The maximum compressive strain in concrete in axial compression is 0.002 at failure.

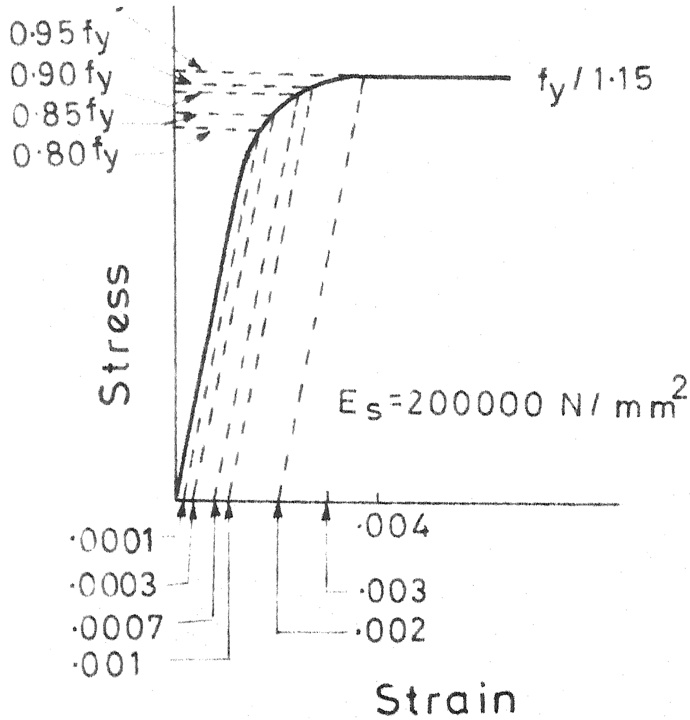


Fig. 3·21(a) Stress-strain curve for cold worked deformed bar

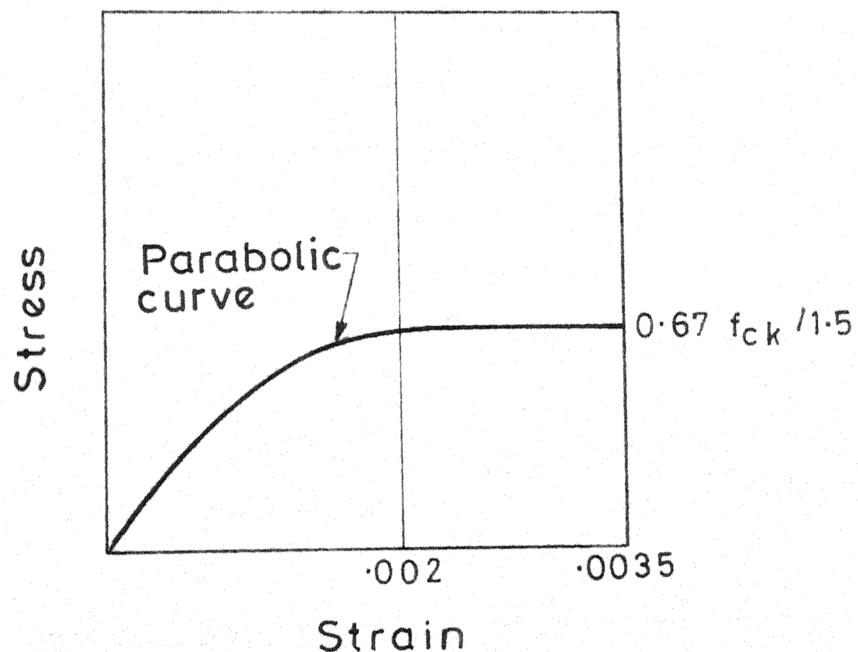


Fig. 3·21 (b) Stress-strain curve for concrete

(g) The maximum strain at the most highly compressed extreme fibre under axial load and bending shall be taken as 0.0035 minus 0.75 times the strain at the least compressed fibre when there is no tension on the section.

3.6 Optimization

Most of the engineering designs turn out to be non-linear constrained optimization problems which can be stated in the standard mathematical form as:

Find vector $\bar{X}$ of the design variables which minimises the objective function $F(\bar{X})$ subject to m constraints $g_j(\bar{X}) \leq 0$, $j = 1, 2, \ldots m$.

The objective function considered in the present study is to minimize the cost of the superstructure of the water tower. The choice of the design variables and the constraints on the design are discussed in the subsequent sections with special reference to certain important aspects of the problem at hand. The precise mathematical programming formulation subsequently follows.

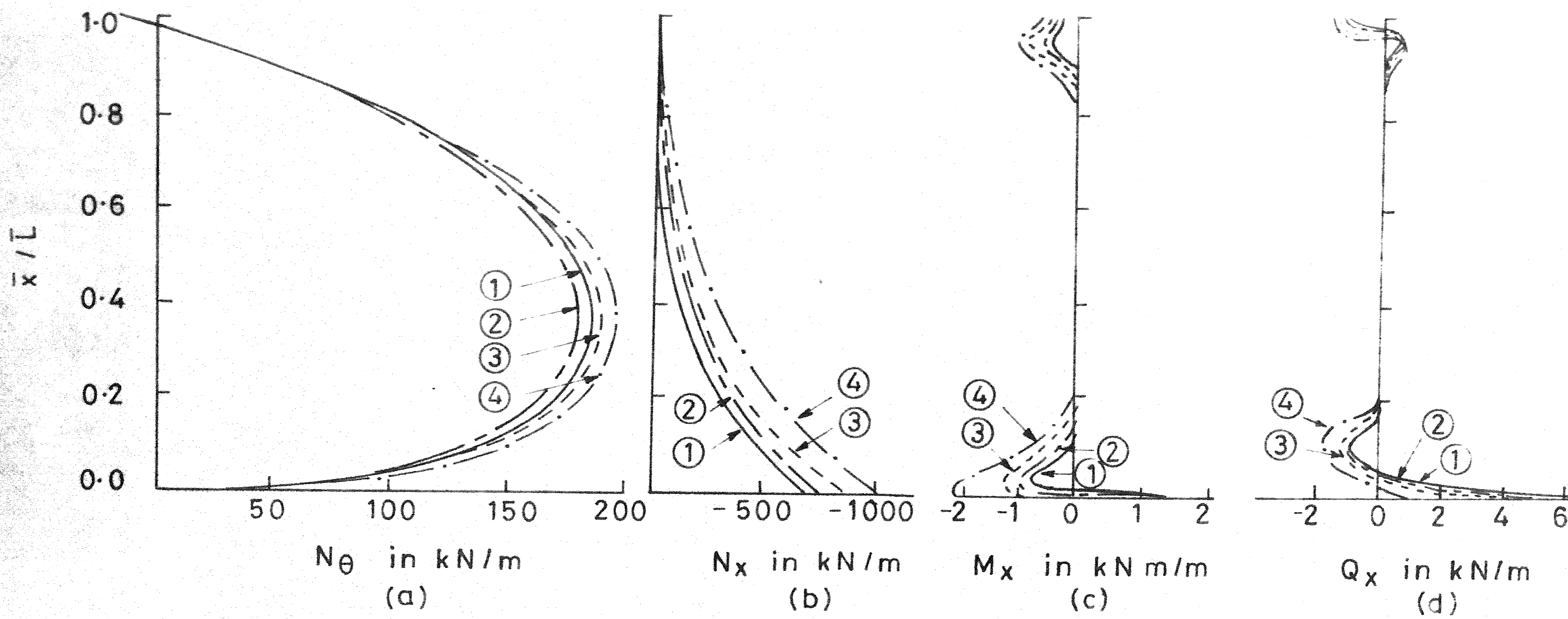
3.6.1 Organisation of optimization process - some practical considerations

Optimization process is a cyclic design-analysis procedure. The computational time to obtain the optimum design increases with the increase in the number of design variables and the number of constraints in the problem. Therefore, a serious thought is required to be given to the choice of design variables and the constraints to be included

in the optimization framework. With the variation in the design variables, the design forces also vary. However, if the change in the design forces is very small with regard to the variations in the design variables, the design forces can be considered to be invariants in the optimization process. In that situation the design forces can be given as a slightly conservative input and this avoids the computation of the design forces in each cycle of optimization. This leads to a substantial saving of computational time in the overall optimization effort. Although such a situation is a rarity, but in the present study it is so and this has been established while observing critically the results of the elastic analysis for finding the design forces in the reservoir portion. In view of the foregoing considerations, certain important aspects to be incorporated in the optimal design process are discussed herein.

The governing design forces in the reservoir portion are obtained from the elastic analysis for axisymmetric loading combination of dead load and water pressure loading under tank full condition. These forces are the functions of material properties and design parameters like, radius of the supporting shaft 'R_2', the angle 'α' and the thicknesses of the various elements of the tower. The influence of these variables on the design forces of the reservoir portion is found to be as under:

(a) Figures 3.22 and 3.23 show the variation of the stress resultants in the reservoir portion with α and R_2 respectively, for a typical tank of 300 kl capacity. It



① α = 1·091 radians or R_1/R_2=5·0

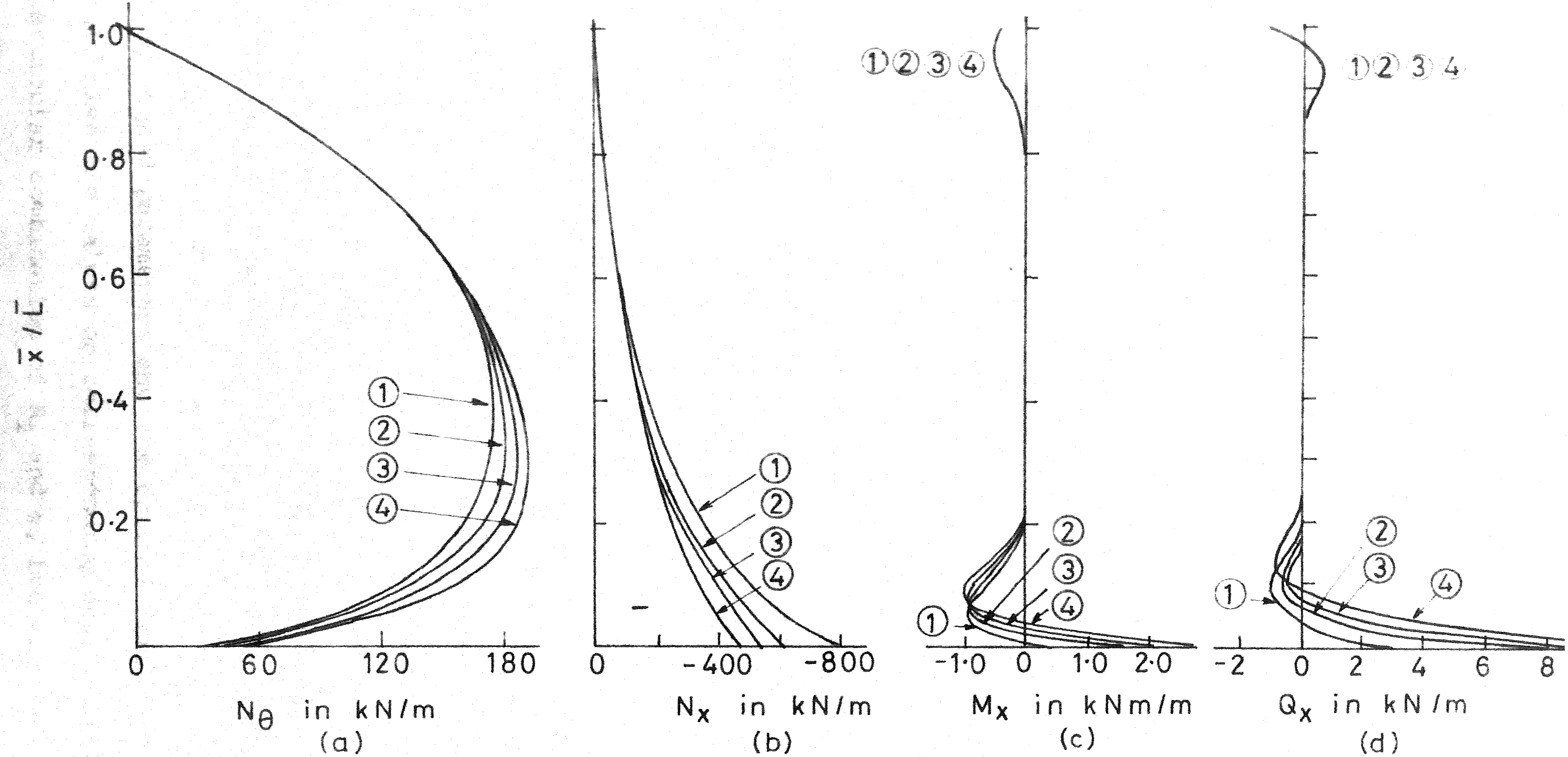
② α = 0·957 " " " =5·5

③ α = 0·822 " " " =6·0

④ α = 0·697 " " " =6·5

Radius of the supporting shaft R_2=1·125 m

Fig.3·22 Variation of stress resultants along the wall of the conical tank with α



① $R_2 = 1{\cdot}125$ m , $R_1/R_2 = 5{\cdot}5$ or $\alpha = 0{\cdot}957$ radian
② $R_2 = 1{\cdot}375$ m , $R_1/R_2 = 4{\cdot}5$ " $\alpha = 0{\cdot}977$ "
③ $R_2 = 1{\cdot}625$ m , $R_1/R_2 = 4{\cdot}0$ " $\alpha = 0{\cdot}924$ "
④ $R_2 = 1{\cdot}875$ m , $R_1/R_2 = 3{\cdot}5$ " $\alpha = 0{\cdot}934$ "

Fig.3·23 Variation of stress resultants along the wall of the coni
tank with R_2

can be seen that the forces change appreciably with change in both the parameters.

(b) For a particular value of R_2 and α there is hardly any change in the governing design forces, particularly that of the hoop tensions, with the change in thicknesses of various elements. Although the bending moment 'M_x' changes slightly which usually does not govern the design, it is limited to edge region only. The Table 3.17 indicates the variation in the governing design forces with thickness of conical tank and the shaft, for a reservoir of capacity 300 kl.

Unlike the reservoir portion, the governing design forces of the supporting shaft vary appreciably not only with R_2 and α but also with the change in the thickness of the shaft.

In the light of the above observations, there can be two possible approaches to achieve the optimal design. One is the direct approach and the other is an indirect one.

In the direct approach R_2 and α could be considered as design variables with other possible variables like thicknesses and reinforcements for different elements of the tower. The optimal design so obtained will directly give the optimal shape parameters, R_2 and α, and corresponding values of the other design variables.

In the indirect approach R_2 and α can be made as preassigned parameters and the optimal design so obtained will give the values of corresponding design variables for a particular combination of R_2 and α, but with a fixed input of

Table 3.17 Variation in governing design forces of the reservoir portion with thickness of conical tank and the supporting shaft

R_2 in m	in radian	Thickness of different elements in m				Maximum value of stress resultants in conical tank					Maximum bendi moment in inr cylinder in kNm/m
						Hoop tension in kN/m			Bending moment in kNm/m		
		t_c	t_s	t_d	t_{cy}	$N_{\theta 1}$	$N_{\theta 2}$	$N_{\theta 3}$	Posi-tive	Nega-tive	
1.125	0.957	0.15	0.15	0.075	0.125	174.304	135.281	87.931	0.201	0.944	1.182
-	-	0.15	0.175	-	-	174.305	135.282	87.931	0.117	0.969	1.284
-	-	0.15	0.20	-	-	174.306	135.282	87.931	0.040	0.991	1.380
-	-	0.175	0.15	-	-	175.632	137.371	90.544	0.066	1.172	1.276
-	-	0.20	0.15	-	-	176.964	139.508	93.025	0.111	1.422	1.361

Note:- t_d and t_{cy} are not changed as for top dome and inner cylinder minimum thicknesses will be adequate (Section 3.4.3) in all the cases.

design forces for the reservoir portion which are slightly on the conservative side. The design forces corresponding to the optimal design so obtained can finally be evaluated for the reservoir portion and the optimal design rechecked for the actual design forces. The difference between the actual design forces and the input design forces is observed to be negligible basically because the membrane forces which govern the design are insensitive to the thickness variations. Therefore, the revision of the optimal design so obtained is never necessitated. In order to obtain the optimal configuration of the tower through the indirect approach, a parametric study for different combinations of R_2 and α is made in the present work. In order to arrive at the optimal configuration atleast nine different sets of R_2 and α combination have been used in the present work in each case.

The Table 3.18 shows the results of optimal designs obtained through the two approaches, described in the foregoing discussion, for the following design data;

Capacity of the tank = 100 m^3

Height of the staging = 20 m

Free board = 150 mm

Wind zone = II (Basic wind pressure = 1.5 kN/m^2)

It can be seen from the results given in Table 3.18 that the optimal designs obtained by the two approaches are almost the same but the total computational time required through indirect approach is very much less than that required by direct approach. Moreover, the indirect approach gives

Table 3.18 Results of optimal design through two approaches

Approach adopted	R_2 in m	R_1/R_2 or α		Optimal design variables					Optimal configuration parameters			Optimal cost in rupees	Comput tional time i minute
		$\frac{R_1}{R_2}$	α in radian	t_c t_s in m	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	R_2 in m	R_1/R_2	α in radian		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
Direct approach	taken as design variables -	-	-	.1267 .1284	.7348 .3547	.5538 .9307	.3594 1.835	0.25 0.25	1.1875	3.799	0.9310	90412	Approx. 45.0
	0.875	4.5		.1285 .2870	.7091 .5078	.5291 1.277	.3195 1.941	0.25 0.25				102212	1.45
		5.0		.1258 .2284	.6904 .6555	.5806 1.902	.3611 1.990	0.25 0.25				96088	1.50
		5.5		.1250 .200	.7478 1.011	.5800 1.990	.4396 1.999	0.25 0.25				95467	1.60
Indirect approach	1.125	3.5		.1297 .1338	.7942 .8990	.5100 1.835	.3200 1.985	0.25 0.25				95666	1.50
		4.0		.1280 .1290	.7209 .4693	.5394 1.377	.3777 1.878	0.25 0.25	1.125	4.0	0.9260	89904	1.35
		4.5		.1277 .1282	.7739 .4195	.6604 .9810	.4600 1.812	0.25 0.25				90922	1.60

Contd...

Table 3.18 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	1.375	3.0		.1250	.7854	.5578	.3868	0.25				97814	2.20
				.1274	.3083	.6388	1.344	0.25					
		3.5		.1284	.7753	.5722	.3795	0.25				96653	1.40
				.1289	.2604	.5577	1.071	0.25					
		4.0		.1269	.8611	.7275	.5038	0.25				99901	1.45
				.1266	.2500	.5297	.8700	0.25					

optimal designs for different combinations of R_2 and α in addition to the one which refers to the optimal configuration. As mentioned earlier, in the present study, the objective function does not include the cost of the foundation, therefore, the optimal diameter so obtained by direct approach may not sometimes suit the foundation conditions for overall economy. In such cases one has the choice to go for an optimal design with a suitable diameter of the supporting shaft, out of those which are obtained in the process of indirect approach. Thus, indirect approach has many advantages and hence is adopted herein. To arrive at the optimal configuration through indirect approach values of R_2 and R_1/R_2 are increased or decreased by 0.25 m and 0.5 respectively. These intervals are found to be satisfactory both for arriving at near exact optimal design and for standardisation of such designs and their formwork to be used.

It is not always necessary to include all the design requirements as constraints in the optimization process. In order to save computational time, only those constraints should be included which are likely to govern the design. The optimal design so obtained can finally be checked for other design requirements and may be modified, if needed. In view of this consideration, the constraints not included in the optimization process, but almost invariably found to satisfy the final optimal design, are as follows:

(i) The constraints related to safety against collapse for reservoir portion as well as the supporting shaft.

(ii) The constraints dealing with limiting surface flexural crack widths in the shaft.

(iii) Some of the constraints which deal with minimum requirement of reinforcement as expressed by the inequalities (3.40), (3.41) and (3.47), which are given in the Section 3.6.4.

It does not seem feasible to carry out optimal design study, in the present case, for all possible combinations of a wind zones and seismic zones defined in the country without prohibitive expenditure of computer time. Moreover, it has been recognised that designs for all such combinations are not indeed required to be carried out because a water tower designed for a particular wind zone may be safe in many seismic zones. For example, a tower of 100 kl capacity and 20 m staging height, designed for wind zone I (basic wind pressure 1 kN/m^2) is found to be safe for all the five seismic zones of India. Therefore, in the present work, optimal designs are carried out for the various wind zones of India and their suitability is established for seismic zones. Thus, the analysis for finding the seismic forces is carried out only at the time of checking the suitability of the final design for a particular seismic zone. This results in considerable saving of computational time.

To summarise the above discussion, the optimal design of the superstructure of the water tower, in the present work, is carried out for different wind zones by performing a parametric study for a set of values of R_2 and α. The optimal

design so obtained is finally checked for its safety and suitability in various seismic zones.

3.6.2 Design variables

The vector $\bar{X}$ gives the various design variables considered in the present design problem, which is expressed as under:

$$\bar{X} = \begin{bmatrix} t_c \\ p_{rc1} \\ p_{rc2} \\ p_{rc3} \\ p_{rc4} \\ t_s \\ p_{rs1} \\ p_{rs2} \\ p_{rs3} \\ p_{rcy} \end{bmatrix}$$

where t_c = thickness of the conical tank wall,

p_{rc1}, p_{rc2} and p_{rc3} are the percentages of hoop reinforcement in the three regions of the conical tank (Figure 3.15),

p_{rc4} = percentage of reinforcement in meridional direction for conical tank, for a short length at junction 2,

t_s = thickness of the supporting shaft,

p_{rs1}, p_{rs2} and p_{rs3} are the percentages of reinforcement in vertical direction in the three regions of the supporting shaft (Figure 3.16), and

p_{rcy} = percentage of reinforcement in x-direction for inner cylinder, for a short length at junction 2.

The values of maximum bending moments, for conical and inner cylindrical portion of the reservoir, occur in the edge region of junction 2. However, these moments reduce at an exponential rate and hence reinforcement p_{rc4} and p_{rcy} need to be provided only for a short length near the junction. For the remaining length of both the elements, a minimum reinforcement of 0.25% has been found to be more than adequate. Any of these lengths is not found to be more than 1 m, hence in computing the objective function the reinforcements p_{rc4} and p_{rcy} are considered for 1 m length only in the two elements.

3.6.3 Objective function

The cost of concrete, reinforcement, formwork required for the water tower including labour but excluding the foundation, is chosen as the objective function, in the present work.

Adopting the design for top dome and inner cylindrical portion as given in Section 3.4.3 and the reinforcement distribution for conical tank and support as mentioned in Section 3.4.4, the expression for objective function can be written as:

$$
\begin{aligned}
F = \; & 2\pi R_d^2(\cos\varnothing_2 - \cos\varnothing_1)(t_d C_c + \frac{2 \times 0.25}{100} t_d \rho_s C_s + C_f) \\
& + \frac{\pi(R_1^2 - R_2^2)}{\cos\alpha}(t_c C_c + \frac{0.25}{100} t_c \rho_s C_s + 2C_f) \\
& + \frac{\pi(R_3^2 - R_2^2)}{\cos\alpha}(\frac{p_{rc1}}{100} t_c \rho_s C_s) + \frac{\pi(R_4^2 - R_3^2)}{\cos\alpha}(\frac{p_{rc2}}{100} t_c \rho_s C_s \\
& + \frac{\pi(R_1^2 - R_4^2)}{\cos\alpha}(\frac{p_{rc3}}{100} t_c \rho_s C_s) + 2\pi R_{cy} L_{cy}(t_{cy} C_c \\
& + \frac{2 \times 0.25}{100} t_{cy} \rho_s C_s + 2C_f) + 2\pi R_2 L \; [t_s C_c + \frac{0.25}{100} t_s \rho_s C_s \\
& + 2C_f + \frac{t_s \rho_s C_s}{300}(p_{rs1} + p_{rs2} + p_{rs3})] \\
& + \frac{\pi[(R_2 + 1.0\cos\alpha)^2 - R_2^2]}{\cos\alpha}(\frac{p_{rc4} - 0.25}{100} t_c \rho_s C_s) \\
& + 2\pi R_{cy}(\frac{p_{rcy} - 0.25}{100} t_{cy} \rho_s C_s)
\end{aligned}
\tag{3.39}
$$

where R_d = radius of the top dome,

R_{cy} = radius of inner cylinder (which is equal to R_2 in this case),

C_c = cost of finished concrete in rupees per m^3,

C_s = cost of steel in rupees per kN,

C_f = cost of formwork in rupees per square meter,

t_d and t_{cy} = thickness of the top dome and the inner cylinder respectively

ρ_s = weight of steel in kN/m^3,

L_{cy} and L = length of inner cylinder and supporting shaft respectively, and all other terms are as defined earlier.

3.6.4 The constraints

To take care of the temperature variations and shrinkage effects, the percentage of reinforcement for deformed bars, in each of the two directions should not be less than $p_{r\,min}$, which is equal to 0.25% as per IS: 3370. Thus, the eight constraints can be expressed in the form

$$\frac{p_{r\,min}}{p_{rc1}} - 1 \leq 0 , \qquad (3.40)$$

$$\frac{p_{r\,min}}{p_{rc2}} - 1 \leq 0 , \qquad (3.41)$$

$$\frac{p_{r\,min}}{p_{rc3}} - 1 \leq 0 , \qquad (3.42)$$

$$\frac{p_{r\,min}}{p_{rc4}} - 1 \leq 0 , \qquad (3.43)$$

$$\frac{p_{r\,min}}{p_{rcy}} - 1 \leq 0 , \qquad (3.44)$$

$$\frac{p_{r\,min}}{p_{rs1}} - 1 \leq 0 , \qquad (3.45)$$

$$\frac{p_{r\,min}}{p_{rs2}} - 1 \leq 0 , \qquad (3.46)$$

and

$$\frac{p_{r\,min}}{p_{rs3}} - 1 \leq 0 . \qquad (3.47)$$

Practical considerations dictate a minimum thickness, t_{min}, for conical tank wall and the cylindrical shaft. There is generally no advantage in adopting a thickness less than 150 mm-125 mm as the precision required in fixing the reinforcement, placing and compacting concrete would involve extra

cost. An error in the displacement of the reinforcement, provided to resist bending moment, will be more serious in thinner walls. A minimum thickness of 125 mm is adopted for both the elements in the design of Type I towers considered in this chapter. Therefore, the ninth and tenth constraints are written in the form

$$\frac{t_{min}}{t_c} - 1 \leq 0 \,, \tag{3.48}$$

and

$$\frac{t_{min}}{t_s} - 1 \leq 0 \,. \tag{3.49}$$

At service loads cracks due to hoop tension in the conical reservoir portion are controlled by limiting the direct tensile stresses in steel as well as concrete to the maximum permissible values, $f_{st\ max}$ and $f_{ct\ max}$, which are 100 MPa and 1.2 MPa for steel and M20 concrete respectively. These checks are to be carried out for each of the three regions of hoop reinforcement. Thus, the constraints eleven through sixteen can be written as

$$(N_{\theta 1})_{max} / [t_c (\frac{p_{rc1}}{100})\ f_{st\ max}] - 1 \leq 0 \,, \tag{3.50}$$

$$(N_{\theta 2})_{max} / [t_c (\frac{p_{rc2}}{100})\ f_{st\ max}] - 1 \leq 0 \,, \tag{3.51}$$

$$(N_{\theta 3})_{max} / [t_c (\frac{p_{rc3}}{100})\ f_{st\ max}] - 1 \leq 0 \,, \tag{3.52}$$

$$\frac{(N_{\theta 1})_{max}}{A_{eu1}\ f_{ct\ max}} - 1 \leq 0 \,, \tag{3.53}$$

$$\frac{(N_{\theta 2})_{max}}{A_{eu2}\ f_{ct\ max}} - 1 \leq 0 , \tag{3.54}$$

and

$$\frac{(N_{\theta 3})_{max}}{A_{eu3}\ f_{ct\ max}} - 1 \leq 0 , \tag{3.55}$$

A_{eui} is the equivalent uncracked sectional area of concrete in ith region of hoop reinforcement which is given by

$$A_{eui} = t_c \left[1 + \frac{(m - 1)p_{rci}}{100}\right]$$

where m = modular ratio.

Serviceability limit state of cracking also demands that flexural tensile stress in concrete, computed on equivalent uncracked section basis, and in steel based on cracked section are less than 1.70 MPa and 100.0 MPa respectively. Maximum bending moments in x-direction for both conical and inner cylindrical shells occur near the edge region at junction 2. However, in conical shell the value of stress resultant N_x will also be maximum at junction 2 which will nullify the bending tensile stress and there will be hardly any need for carrying out the check for flexural cracks. Whereas in case of inner cylinder N_x is caused only due to self weight of the cylinder and for all practical purposes it is neglected. If M_{cyc} and M_{cys} are the maximum bending moment carrying capacities of the section of unit width for the inner cylinder, as per the limiting stresses defined in the foregoing for concrete and steel respectively. Then the constraints seventeen and eighteen can be expressed in the form

$$\frac{M_{cya}}{M_{cyc}} - 1 \leq 0 , \tag{3.56}$$

and

$$\frac{M_{cya}}{M_{cys}} - 1 \leq 0 . \tag{3.57}$$

where M_{cya} = actual maximum bending moment in the edge region at junction 2 for unit width along circumference of the inner cylinder, at service loads.

The limit analysis, carried out to assess the strength of the tank to resist the water pressure, furnishes the value of collapse pressure p_o. Using a partial safety factor 1.5 against collapse, the nineteenth constraint can be written as

$$\frac{1.5 \rho (x_2 - x_1) \sin\alpha}{p_o} - 1 \leq 0 \tag{3.58}$$

If M_{urs} is the ultimate moment of resistance of the circular shaft at its critical section under combined axial and lateral loads at collapse and M_{uas} is the actual value of the bending moment at that section, then the safety against collapse demands that $M_{urs} \geq M_{uas}$. Thus, the twentieth constraint may be stated as

$$\frac{M_{uas}}{M_{urs}} - 1 \leq 0 \tag{3.59}$$

Although the collapse in real sense will occur when maximum water pressure in the conical tank reaches the value p_o or the maximum bending moment at the critical section of

the shaft becomes equal to M_{urs}, but the structure will become unserviceable as soon as the first hinge circle forms in the conical portion or considering an element of unit width, the ultimate limit state of strength of a local section of the shaft is reached. Usually the limit state of collapse will not govern the design of the tower. Whereas the other two cases of failure, due to ultimate limit state of strength being reached at local sections, may be critical in some cases. Hence, safety against such failures is more important. The first hinge circle in the conical tank will be formed in the edge region of junction 2 because both M_x and N_x are higher in this region. Whereas the critical sections for the shaft will be one just above the foundation and other two at points of curtailment of vertical reinforcement. If M_{urc} is the ultimate moment of resistance per unit width of the conical shell at the critical section in presence of axial thrust and M_{uac} is the actual value of maximum bending moment at that section, then for safety $M_{urc} \geq M_{uac}$. Thus the twentyfirst constraint can be written as

$$\frac{M_{uac}}{M_{urc}} - 1 \leq 0 \qquad (3.60)$$

Under dead load + water load combination, when tank is full, the shaft is subjected to axial compressive stresses. For other loading combinations also, due to combined action of axial and lateral forces, a local section of supporting cylindrical shaft of unit width will have almost uniform axial strain across the thickness of the shell. Thus, the

ultimate strength in axial compression, P_{ursl}, P_{urs2} and P_{urs3} of such local sections is required to be greater than the actual maximum ultimate forces, P_{uasl}, P_{uas2} and P_{uas3}, coming on to these sections. Hence the constraints twenty-two through twenty-four can be written as

$$\frac{P_{uasl}}{P_{ursl}} - 1 \leq 0 , \tag{3.61}$$

$$\frac{P_{uas2}}{P_{urs2}} - 1 \leq 0 , \tag{3.62}$$

and

$$\frac{P_{uas3}}{P_{urs3}} - 1 \leq 0 . \tag{3.63}$$

The constraints twenty-five through twenty-seven dealing with surface flexural crack widths, w_{cr1}, w_{cr2} and w_{cr3}, due to maximum bending moments in the three regions of vertical reinforcement of the supporting shaft, can be expressed in the form

$$\frac{w_{cr1}}{w_{cra}} - 1 \leq 0 , \tag{3.64}$$

$$\frac{w_{cr2}}{w_{cra}} - 1 \leq 0 , \tag{3.65}$$

and

$$\frac{w_{cr3}}{w_{cra}} - 1 \leq 0 . \tag{3.66}$$

where w_{cra} = the maximum allowable width of crack, which is taken as 0.3 mm.

As suggested in Section 3.4.2, the procedure for cracks control should be used as a guide line because its reliability is still being debated. Therefore an indirect

way of crack control by limiting the maximum deflection still has its place in the design procedures. Moreover, deflection of the supporting shaft, due to lateral loads, results in additional moments induced by vertical loads. If the deflection is not restricted, these moments may considerably affect the design of the shaft and the foundation and thereby the overall economy. Usually for tower-type engineering installations the maximum allowable deflection is taken between $\frac{1}{500}$ to $\frac{1}{400}$ of the height. However in the present study the maximum allowable deflection at the top of the shaft is taken as $\frac{1}{450}$ of the height of the shaft. Thus, the twenty-eighth constraint can be written as

$$\frac{\delta_{act}}{\delta_{all}} - 1 \leq 0 . \tag{3.67}$$

where δ_{act} = actual deflection at the top of the shaft, and

δ_{all} = maximum allowable deflection.

To safeguard against elastic instability of the supporting shaft, the procedure given in IS: 2210 is adopted in the present study. For safety the actual compressive stress, f_{act}, at service loads should not exceed the permissible critical stress, f_{crit}. Thus, the twenty-ninth constraint is given as

$$\frac{f_{act}}{f_{crit}} - 1 \leq 0 \tag{3.68}$$

Although the amount of wind induced ring moment in the shaft is small and does not demand any constraint for safety and serviceability, but for completeness the thirtieth

constraint dealing with the surface flexural crack width, w_{cr4}, can be written as

$$\frac{w_{cr4}}{w_{cra}} - 1 \leq 0 \tag{3.69}$$

3.6.5 Optimal design formulation and solution procedure

From the foregoing discussion it can be concluded that the formulated optimal design problem to obtain the optimal configuration and the design of the water tower turns out to be a problem of ten design variables, as defined in Section 3.6.2, and having thirty constraints as described in Section 3.6.4. The objective function defined in the Section 3.6.3 is a non-linear function of the design variables and so also most of the constraint functions. Therefore, the formulated optimal design problem is a non-linear constrained programming problem which has been converted into unconstrained non-linear programming problem through the use of interior penalty function. The unconstrained non-linear programming problem so obtained is finally solved as a sequence of unconstrained minimization problems by Davidon-Fletcher-Powell method as outlined in Section 1.4.5.

3.7 Parametric Study

Optimal limit state design of Type I water towers has been carried out for the following cases:

(1) Capacity in m^3: 100, 200 and 300.

(2) Staging height in m: 15, 20 and 25.

(3) Basic wind pressure in kN/m^2: 1.0, 1.5 and 2.0.

The suitability of these designs, obtained for various wind zones, is established for the different seismic zones. A free board of approximately 150 mm has been provided in all cases.

3.8 Rates of Construction Materials

Current rates of the different construction materials, which are used for the evaluation of the cost of the superstructure of the tower are given as under:

Cost of finished cement concrete of grade M20	=	Rs. 750 per m^3
Cost of high yield deformed bars including cost of cutting, bending and laying	=	Rs. 600 per kN
Cost of formwork	=	Rs. 600 per m^2

In order to investigate the effect of escalation in the material cost, the practice of using cost ratio of materials is followed. Cost of one m^3 finished concrete and one m^2 of formwork have been normalized with cost of one kN of reinforcement and the resulting cost ratios are termed CR1 and CR2 respectively. Thus, the values for CR1 and CR2 as per current rates are 1.25 and 0.10 respectively.

3.9 Results and Discussions

The optimal design results have been presented essentially in the tabular form. For a given capacity of tank, staging height, seismic and wind zones an optimal design can be picked from the design Tables 3.19 through 3.21 which gives both optimal configuration and design variables for the tower.

Table 3.19 Optimal designs of Type I towers with 100 m^3 capacity tank

L (m)	Wind zone p_b (kN/m^2)	Optimal design variables					Optimal configuration parameters				Optimal cost (Rs)	Applicable to seismic zones
		t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	R_2 (m)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
25	2.0	125 126	0.834 0.314	0.574 0.967	0.388 1.788	0.35 0.35	1.625	3.0	0.872	5.055	126212	I-V
	1.5	128 132	0.782 0.372	0.569 1.070	0.380 1.910	0.25 0.25	1.375	3.5	0.852	5.223	115414	I-V
	1.0	128 147	0.726 0.374	0.527 1.260	0.414 1.907	0.25 0.25	1.125	4.0	0.926	5.615	104523	I-V
20	2.0	125 128	0.834 0.294	0.650 0.867	0.420 1.530	0.25 0.25	1.375	3.5	0.852	5.223	99196	I-V
	1.5	128 129	0.721 0.469	0.539 1.378	0.377 1.879	0.25 0.25	1.125	4.0	0.926	5.615	89904	I-V
	1.0	128 156	0.718 0.409	0.580 1.324	0.399 1.927	0.25 0.25	0.875	5.5	0.788	5.582	83019	I-V
15	2.0	125 125	0.720 0.289	0.546 0.720	0.364 1.627	0.25 0.25	1.125	4.0	0.926	5.615	75732	I-V
	1.5	125 139	0.732 0.360	0.608 1.091	0.420 1.804	0.25 0.25	0.875	5.5	0.788	5.582	70733	I-V
	1.0	125 125	0.735 0.287	0.598 0.679	0.424 1.360	0.25 0.25	0.875	5.5	0.788	5.582	67735	I-V

Table 3.20 Optimal designs of Type I towers with 200 m^3 capacity tank

L (m)	Wind zone p_o (kN/m^2)	Optimal design variables: t_c / t_s (mm)	p_{rc1} / p_{rs1}	p_{rc2} / p_{rs2}	p_{rc3} / p_{rs3}	p_{rc4} / p_{rcy}	Optimal configuration parameters: R_2 (m)	R_1/R_2	α (rad)	$\bar{L}$ (m)	Optimal cost (Rs)	Applicable to seismic zones
25	2.0	130 140	1.145 0.814	0.836 1.553	0.701 1.930	0.50 0.55	1.625	3.5	0.946	6.944	166189	I–V
	1.5	125 155	1.213 0.603	0.987 1.405	0.692 1.928	0.25 0.25	1.375	4.5	0.779	6.764	153748	I–V
	1.0	126 125	1.196 0.362	0.938 0.948	0.674 1.666	0.25 0.25	1.375	4.5	0.779	6.764	140837	I–V
20	2.0	127 138	1.187 0.376	0.914 1.117	0.627 1.906	0.25 0.25	1.375	4.5	0.779	6.764	132777	I–V
	1.5	128 184	1.093 0.565	0.912 1.249	0.656 1.908	0.25 0.25	1.125	5.0	0.908	7.317	126791	I–V
	1.0	127 130	1.111 0.436	0.835 1.206	0.563 1.873	0.25 0.25	1.125	5.0	0.908	7.317	115442	I–V
15	2.0	126 148	1.088 0.430	0.933 1.242	0.619 1.785	0.25 0.25	1.125	5.0	0.908	7.317	106826	I–V
	1.5	129 128	1.065 0.414	0.871 0.840	0.572 1.596	0.25 0.25	1.125	5.0	0.908	7.317	103121	I–V
	1.0	128 128	1.106 0.300	0.879 0.485	0.613 1.013	0.25 0.25	1.125	5.0	0.908	7.317	101418	I–IV

Table 3.21 Optimal designs of Type I towers with 300 m^3 capacity tank

L (m)	Wind zone p_b (kN/m^2)	Optimal design variables					Optimal configuration parameters				Optimal cost (Rs)	Applicable to seismic zones
		t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{ray}	R_2 (m)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
25	2.0	135 127	1.440 0.455	1.018 1.174	0.722 1.971	0.70 0.80	1.875	3.5	0.934	7.889	200022	I-V
	1.5	132 143	1.435 0.445	1.090 1.233	0.676 1.888	0.50 0.60	1.625	4.0	0.924	8.087	186975	I-V
	1.0	131 135	1.450 0.440	1.136 1.129	0.827 1.999	0.25 0.25	1.375	5.0	0.811	7.988	170550	I-V
20	2.0	131 127	1.416 0.434	1.118 1.079	0.717 1.809	0.50 0.60	1.625	4.0	0.924	8.087	164292	I-V
	1.5	127 145	1.450 0.548	1.100 1.204	0.571 1.845	0.45 0.35	1.375	4.5	0.977	8.592	155619	I-V
	1.0	125 156	1.430 0.561	1.122 1.427	0.738 1.940	0.25 0.25	1.125	5.5	0.957	8.785	143787	I-V
15	2.0	126 184	1.436 0.594	1.143 1.415	0.575 1.850	0.25 0.25	1.125	5.5	0.957	8.785	135893	I-V
	1.5	125 155	1.410 0.354	1.103 1.141	0.705 1.842	0.25 0.25	1.125	5.5	0.957	8.785	129354	I-V
	1.0	128 152	1.396 0.273	1.082 0.652	0.711 1.139	0.25 0.25	1.125	5.5	0.957	8.785	126394	I-IV

The Figure 3.22 shows the variation of stress resultants along the wall of the conical tank with α for the tank of 300 kl capacity which is supported on the shaft of 1.125 m radius. It can be seen that for $\alpha = 0.957$ radian the maximum hoop tension and bending moment are minimum and the shearing force Q_x also reduces considerably. The Table 3.21 gives optimal designs for 300 kl capacity tank. It is interesting to note that for some cases of wind zones and heights of shaft the optimal configuration also turns out to be $\alpha = 0.957$ radian and $R_2 = 1.125$ m. Thus, the results of the Figure 3.22 and Table 3.21 together indicate that the optimal configuration of the reservoir adjusts itself in a manner that the bending moment and shearing forces at junctions reduce considerably bringing the state of stress in the reservoir portion in the close proximity to that of the membrane state. It is also observed that the value of maximum hoop tension in the reservoir becomes minimum for the optimal configuration, in most of the cases.

The Table 3.25 indicates the effect of cost ratios on optimal configuration. It can be seen from the results that for the various combinations of cost ratios CR1 and CR2, the optimal configuration comes out to be the same. It is also observed that for a particular shaft radius the optimal value of R_1/R_2 remains the same for different cost ratio combinations. The Table 3.26 shows the effect of cost ratio on optimal design variables. There is hardly any change in the design variables of the reservoir portion but the design

Table 3.22 . Optimal designs of Type I towers with 100 m^3 capacity tank for non-optimal shaft radii

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	Optimal design variables: t_c / t_s (mm)	p_{rc1} / p_{rs1}	p_{rc2} / p_{rs2}	p_{rc3} / p_{rs3}	p_{rc4} / p_{rcy}	Optimal configuration parameters: R_1/R_2	α (rad)	$\bar{L}$ (m)	Optimal cost (Rs)	Applicable to seismic zones
1	2	3	4	5	6	7	8	9	10	11	12	13
25	2.0	1.125	125 305	0.725 0.309	0.540 1.245	0.370 1.861	0.25 0.25	4.0	0.926	5.615	141564	I-V
		1.375	129 154	0.801 0.965	0.550 1.497	0.380 1.928	0.25 0.25	3.5	0.852	5.223	128401	I-V
	1.5	1.125	126 190	0.737 0.611	0.567 1.931	0.384 1.990	0.25 0.25	4.0	0.926	5.615	122067	I-V
		1.625	128 129	0.815 0.252	0.585 0.620	0.395 1.220	0.35 0.35	3.0	0.872	5.055	122625	I-V
	1.0	1.375	125 128	0.767 0.250	0.573 0.596	0.415 1.273	0.25 0.25	3.5	0.852	5.223	108616	I-V
		0.875	129 253	0.736 0.485	0.589 1.444	0.407 1.918	0.25 0.25	5.5	0.788	5.582	112377	I-V
20	2.0	1.125	128 188	0.704 0.458	0.534 1.174	0.350 1.695	0.25 0.25	4.0	0.926	5.615	100158	I-V
		1.625	126 128	0.841 0.255	0.573 0.469	0.390 0.922	0.35 0.35	3.0	0.872	5.055	106431	I-V

Contd...

Table 3.22 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
	1.5	0.875	125 225	0.747 0.655	0.589 1.930	0.439 1.990	0.25 0.25	5.5	0.788	5.582	95467	I-V
		1.375	128 129	0.775 0.260	0.572 0.558	0.379 1.071	0.25 0.25	3.5	0.852	5.223	96653	I-V
	1.0	1.125	127 128	0.708 0.276	0.557 0.615	0.361 1.407	0.25 0.25	4.0	0.926	5.615	85432	I-V
		1.375	127 125	0.773 0.255	0.572 0.358	0.379 0.663	0.25 0.25	3.5	0.852	5.223	93659	I-V
15	2.0	0.875	127 182	0.725 0.526	0.604 1.008	0.425 1.806	0.25 0.25	5.5	0.788	5.582	75905	I-V
		1.375	127 127	0.769 0.250	0.585 0.351	0.386 0.677	0.25 0.25	3.5	0.852	5.223	82906	I-V
	1.5	1.125	127 126	0.721 0.250	0.541 0.512	0.365 1.029	0.25 0.25	4.0	0.926	5.615	74424	I-V
	1.0	1.125	125 126	0.743 0.263	0.550 0.380	0.370 0.677	0.25 0.25	4.0	0.925	5.615	73500	I-V

Table 3.23 Optimal designs of Type I towers with 200 m^3 capacity tank for non-optimal shaft radii

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	Optimal design variables: t_c / t_s (mm)	p_{rc1} / p_{rs1}	p_{rc2} / p_{rs2}	p_{rc3} / p_{rs3}	p_{rc4} / p_{rcy}	Optimal configuration parameters: R_1/R_2	α (rad)	$\bar{L}$ (m)	Optimal cost (Rs)	Applicable to seismic zones
1	2	3	4	5	6	7	8	9	10	11	12	13
25	2.0	1.375	128 200	1.182 0.418	0.922 1.737	0.617 1.955	0.25 0.25	4.5	0.779	6.764	170022	I-V
		1.875	126 130	1.275 0.270	0.892 0.946	0.565 1.172	0.80 0.85	3.0	0.998	6.900	169953	I-V
	1.5	1.125	128 302	1.102 0.449	0.898 1.266	0.616 1.862	0.25 0.25	5.0	0.908	7.317	168120	I-V
		1.625	128 125	1.145 0.315	0.836 1.150	0.697 1.818	0.50 0.55	3.5	0.946	6.944	155139	I-V
	1.0	1.125	128 211	1.109 0.455	0.899 1.124	0.565 1.754	0.25 0.25	5.0	0.908	7.317	145522	I-V
		1.625	128 130	1.145 0.326	0.836 0.572	0.697 1.109	0.50 0.55	3.5	0.946	6.944	14995	I-V
20	2.0	1.125	125 240	1.140 0.489	0.918 1.417	0.574 1.922	0.25 0.25	5.0	0.908	7.317	138293	I-V
		1.625	127 127	1.204 0.312	0.885 0.832	0.665 1.339	0.50 0.55	3.5	0.946	6.944	137288	I-V

Contd...

Table 3.23 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
	1.5	1.375	127	1.182	0.922	0.629	0.25	4.5	0.779	6.764	128246	I-V
			129	0.317	0.925	1.396	0.25					
		1.625	128	1.149	0.875	0.665	0.50	3.5	0.946	6.944	134446	I-V
			128	0.255	0.509	0.983	0.55					
	1.0	1.375	125	0.120	0.936	0.645	0.25	4.5	0.779	6.764	123899	I-V
			128	0.255	0.490	0.865	0.25					
		1.625	126	1.200	0.886	0.703	0.50	3.5	0.946	6.944	131958	I-IV
			128	0.260	0.317	0.675	0.55					
15	2.0	1.375	125	1.204	0.937	0.648	0.25	4.5	0.779	6.764	112841	I-V
			127	0.308	0.548	0.964	0.25					
		1.625	126	1.180	0.877	0.582	0.50	3.5	0.946	6.944	118779	I-V
			125	0.250	0.356	0.738	0.55					
	1.5	1.375	125	1.195	0.932	0.635	0.25	4.5	0.779	6.764	111450	I-IV
			127	0.250	0.363	0.739	0.25					
	1.0	1.375	125	1.211	0.937	0.648	0.25	4.5	0.779	6.764	110582	I-III
			126	0.250	0.275	0.380	0.25					

Table 3.24 Optimal designs of Type I towers with 300 m^3 capacity tank for non-optimal shaft radii

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	Optimal design variables					Optimal configuration parameters			Optimal cost (Rs)	Applicable to seismic zones
			t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	R_1/R_2	α (rad)	$\bar{L}$ (m)		
1	2	3	4	5	6	7	8	9	10	11	12	13
25	2.0	1.625	132 196	1.404 0.389	1.067 1.026	0.692 1.902	0.50 0.60	4.0	0.924	8.087	203385	I-V
	1.5	1.375	131 192	1.440 0.533	1.090 1.384	0.750 1.946	0.25 0.25	5.0	0.811	7.988	190445	I-V
		1.875	135 129	1.465 0.312	1.096 0.656	0.722 1.540	0.70 0.80	3.5	0.934	7.889	194895	I-V
	1.0	1.125	127 251	1.420 0.521	1.111 1.333	0.714 1.910	0.25 0.25	5.5	0.957	8.785	180106	I-V
		1.625	132 128	1.407 0.284	1.067 0.719	0.694 1.390	0.50 0.60	4.0	0.924	8.087	175377	I-V
20	2.0	1.375	131 165	1.426 0.585	1.097 1.227	0.751 1.890	0.25 0.25	5.0	0.811	7.988	164918	I-V
		1.875	135 129	1.444 0.273	1.047 0.606	0.692 1.085	0.70 0.80	3.5	0.934	7.889	175366	I-V

Contd...

Table 3.24 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13
	1.5	1.125	129 209	1.430 0.577	1.149 1.418	0.777 1.946	0.25 0.25	6.0	0.811	8.267	159147	I-V
		1.625	132 131	1.449 0.333	1.067 0.542	0.703 1.343	0.50 0.60	4.0	0.924	8.087	161133	I-V
	1.0	1.375	128 132	1.411 0.326	1.059 0.738	0.706 1.385	0.45 0.35	4.5	0.977	8.592	148041	I-V
		1.625	131 125	1.467 0.265	1.068 0.443	0.695 0.755	0.25 0.25	4.0	0.924	8.087	157068	I-IV
15	2.0	1.375	130 128	1.448 0.377	1.142 0.718	0.760 1.446	0.25 0.25	5.0	0.811	7.988	139953	I-V
		1.625	133 126	1.406 0.255	1.034 0.472	0.672 0.952	0.50 0.60	4.0	0.924	8.087	143400	I-IV
	1.5	1.375	129 128	1.413 0.327	1.068 0.610	0.676 1.195	0.45 0.35	4.5	0.977	8.592	134334	I-IV
	1.0	1.375	127 128	1.411 0.303	1.080 0.445	0.735 0.681	0.45 0.35	4.5	0.977	8.592	133247	I-III

Table 3.25 Effect of cost ratios on optimal configuration

Capacity of tank = 300 m^3, L = 25 m, p_b = 1.5 kN/m^2

S. No.	CR1	CR2	R_2 (m)	R_1/R_2	Optimal cost (Rs)	Optimal configuration
1	1.25	0.10	1.375	4.5	194073	
				5.0	190509	
				5.5	196318	
			1.625	3.5	192113	R_2 = 1.625 m,
				4.0	186975	R_1/R_2 = 4.0 or
				4.5	190509	
			1.875	3.0	208645	α = 0.924 rad
				3.5	194895	
				4.0	198848	
2	1.75	0.10	1.375	4.5	223060	
				5.0	218856	
				5.5	225521	
			1.625	3.5	222096	R_2 = 1.625 m,
				4.0	211467	R_1/R_2 = 4.0 or
				4.5	218041	
			1.875	3.0	237973	α = 0.924 rad
				3.5	222587	
				4.0	229977	
3	1.25	0.15	1.375	4.5	227892	
				5.0	225197	
				5.5	233870	
			1.625	3.5	228795	R_2 = 1.625 m,
				4.0	224301	R_1/R_2 = 4.0 or
				4.5	230481	
			1.875	3.0	248978	α = 0.924 rad
				3.5	235343	
				4.0	240764	

Table 3.26 Effect of cost ratios on optimal design variables

Capacity of tank = 300 m^3, L = 25 m, p_b = 1.5 kN/m^2

S. No.	R_2 (m)	R_1/R_2	CR1	CR2	Optimal design variables					Optimal cost (Rs)
					t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	
1	1.625	4.0	1.25	0.10	132	1.435	1.090	0.676	0.25	186975
					143	0.445	1.233	1.888	0.25	
2	1.625	4.0	1.75	0.10	131	1.444	1.080	0.690	0.25	211467
					135	0.610	1.309	1.865	0.25	
3	1.625	4.0	1.25	0.15	133	1.390	1.060	0.695	0.25	224301
					143	0.445	1.235	1.886	0.25	
4	1.625	4.0	0.75	0.10	133	1.434	1.047	0.700	0.25	160955
					145	0.429	1.096	1.858	0.25	

variables of the supporting shaft change slightly with the change in the cost of concrete. It is observed that with the increase in cost of concrete, for cost of formwork remaining the same, the thickness of the supporting shaft decreases and amount of the reinforcement increases slightly. Whereas for a particular cost ratio of concrete to steel the design variables are not found to be sensitive to any change in the cost of formwork. Thus, for a specified cost ratio of concrete to steel, it is observed that both the optimal configuration and design variables are not sensitive to changes in the cost of formwork. On the other hand for various cost ratios of concrete to steel, only the optimal design variables of the supporting shaft do vary but only marginally, while the optimal configuration remains essentially unaltered.

It is clear from the foregoing discussion that changes in costs of different materials will not alter the optimal configuration of the tower and the design variables of the reservoir portion and only the design variables of the supporting shaft change very slightly with the change in the value of CR1. Thus for all practical purposes the optimal design obtained using absolute costs of materials prevailing at the time of investigation will also be optimal or very near to the optimal design when the costs of materials are different.

The design Tables 3.19 through 3.21 show that in case of a particular capacity of tank, for more than one wind zone and heights of staging, the optimal configuration comes out to be same. In such cases it is expected that the

design variables of the reservoir portion also remain the same as the governing design forces of the reservoir do not change with changes in wind forces and the staging heights. However, a little variation found in these variables is unavoidable in solutions based upon any numerical technique. Thus, while standardising these designs, this particular fact is to be kept in view.

In some of the cases, thicknesses of the conical tank and the supporting shaft have assumed the minimum value. In optimization studies, when such a situation is encountered, it is normal practice to explore whether the specified minimum can be further reduced. In the present case, since a minimum nominal cover has to be provided and the value of 125 mm has been arrived after considering the limitations imposed by the construction aspects, it is not desirable to reduce the specified minimum thickness any further.

From design Tables 3.19 through 3.24 it can be observed that for small capacity tank the seismic forces are very small as compared to the forces due to wind and the value of seismic forces increases considerably with the increase in capacity. For 100 kl capacity tank the design obtained for wind zone I itself is very safe even for seismic zone V, for all staging heights. Whereas, in case of 300 kl capacity tank, seismic forces in zone IV are comparable to the wind forces of zone I for 15 m staging height. It is also observed that for a given capacity of tank, wind and seismic zones, the ratio of the seismic forces to wind forces increases with

decrease in staging height. Thus, in some cases, for small staging height, the seismic forces may govern the design of the supporting structure when the capacity is more.

The design Tables 3.22 through 3.24 give optimal designs for a radius of supporting shaft which is in the neighbourhood of the optimal radius. These designs are in addition to those corresponding to the optimal radius and may come out to be economical, for a particular foundation condition, when the cost of foundation is also taken into account.

It is interesting to note that the optimal value of α lies between 0.779 to 0.977 radians for all the cases and between 0.85 to 0.96 radians for most of these cases.

The forces in the ring beams at junctions 1 and 2 are of such order that 200 x 200 mm ring beams with 0.50 percent hoop reinforcement is found to be more than adequate for all the cases.

Constraints which are critically satisfied are those regarding the following:

(1) minimum thickness of tank wall,

(2) minimum thickness of supporting shaft for some of the cases,

(3) minimum value of reinforcement in x-direction of conical tank,

(4) maximum values of stress in the hoop reinforcements in all the three regions of the conical tank,

(5) maximum value of direct tensile stress in concrete in the region of maximum hoop tension in conical tank,

(6) maximum deflection of the tower, and

(7) maximum ultimate resistance of a local section of unit width of supporting shaft at its bottom in case of 300 kl capacity tank.

The resistance of the structure to collapse loads is found out to be more than adequate, in all the cases.

Thus the serviceability limit state of cracking is the most critical one in case of reservoir portion, while limit states of deflection and ultimate strength of a local section of the supporting shaft govern its design.

3.10 Method of Using the Design Tables

3.10.1 Example

The method of using the design tables can be illustrated by the following example:

> A conical water tank supported on a cylindrical shaft, is required to be designed for a capacity of 300 m^3. The staging height is 25 m and the wind and seismic zones to be considered are II and IV respectively.

The Table 3.21 is the relevant design table for 300 m^3 capacity tank. It gives optimal radius of the supporting shaft equal to 1.625 m, optimal value of $R_1/R_2 = 4.0$ and the length of the tank wall $\bar{L} = 8.087$ m. The other design variables are as follows:

Thickness of the tank wall 't_c' = 132 mm

Areas of hoop reinforcement in the three regions (Figure 3.15) of the conical portion (as calculated from the percentages of

reinforcement given in the design table) are 1894, 1438 and 892 mm^2/m.

In the conical portion a minimum reinforcement = 330 mm^2/m, in the meridional direction, is found to be adequate for all the cases except for a short length of 1 m near junction 2 in which the reinforcement (as per the percentage of reinforcement given by 'p_{rc4}') for each case will be different. For this case it is = 660 mm^2/m.

Thickness of the supporting shaft 't_s' = 143 mm

Areas of vertical reinforcement in the three regions of supporting shaft (Figure 3.16) are 636, 1763 and 2699 mm^2/m.

Area of circumferential reinforcement for supporting shaft corresponds to the minimum is equal to 357.5 mm^2/m.

A 75 mm thick top dome with minimum reinforcement of 187.5 mm^2/m in both the directions will be adequate for all the cases. Whereas the inner cylindrical shaft of 125 mm thickness with minimum reinforcement = 312.5 mm^2/m, in both the directions, is found to be adequate for all the cases except for a short length of 1 m near junction 2 in which the vertical reinforcement for each case will be different. For this case it is = 750 mm^2/m.

Ring beams of 200 x 200 mm size with 0.50% hoop reinforcement are found to be more than adequate at both the junctions, for all the cases.

CHAPTER 4

OPTIMAL DESIGN OF TYPE II WATER TOWERS

4.1 Design Considerations

For 500 to 1000 kl capacity tanks, water towers of the type shown in Figure 1.1(b) are preferred. In this type of towers, conical tank is provided with a spherical bottom and the diameter of the inner cylinder is kept smaller than that of the supporting shaft. This arrangement results in more space for storage of water as compared to Type I water towers and hence leads to overall economy. The diameter of the inner cylinder is fixed from the consideration of minimum space required for an easy access and in the present study it is kept equal to 1.5 m.

The size of the opening of the bottom dome is quite large as compared to its overall dimension as a result of which the influence of edge-loadings at two ends interact and both membrane and bending forces are found throughout the dome. Although the semi-central angle of this dome can be chosen as design variable, its cost in relation to the cost of the system is very small. Therefore, a suitable value which minimizes the design forces in the dome is obtained indirectly. It is found by actual analysis that the magnitudes of bending moments become minimal for $\phi_1 \simeq 45°$ (ϕ_1 as defined in Figure 3.6), in most of the cases. Hence a value of 45° is chosen for ϕ_1 for the bottom dome. As the size of the bottom dome is small and that too with a large opening

at the top, the whole dome becomes a critical region wherein stress variations are violent. Keeping this in view, as a precautionary measure, minimum thickness of the dome is restricted to 150 mm with reinforcement not less than 0.5% in both the directions. The reinforcement is equally distributed on each face in a staggered fashion. In addition to these, all the design considerations discussed in Section 3.4 will also be applicable to the design of these towers. However, the thickness of the top dome is kept 100 mm, instead of 75 mm as the design forces will be more because of the larger size of the dome as compared to that required for Type I towers.

4.2 Elastic Analysis

In order to find out the design forces in the reservoir portion, elastic analysis is carried out using the force method of analysis already described in Section 3.3.1. The Figure 4.1 shows the water tower under consideration, whereas Figure 4.2 shows the same, separated into simple shell elements with corrective loadings at various junctions.

The compatibility of deformations and equilibrium conditions at junction 1 yield an equation which will be exactly the same as given by the Eq. (3.16). Whereas, for junction 2, these conditions will result in an equation of the form given by Eq. (4.1).

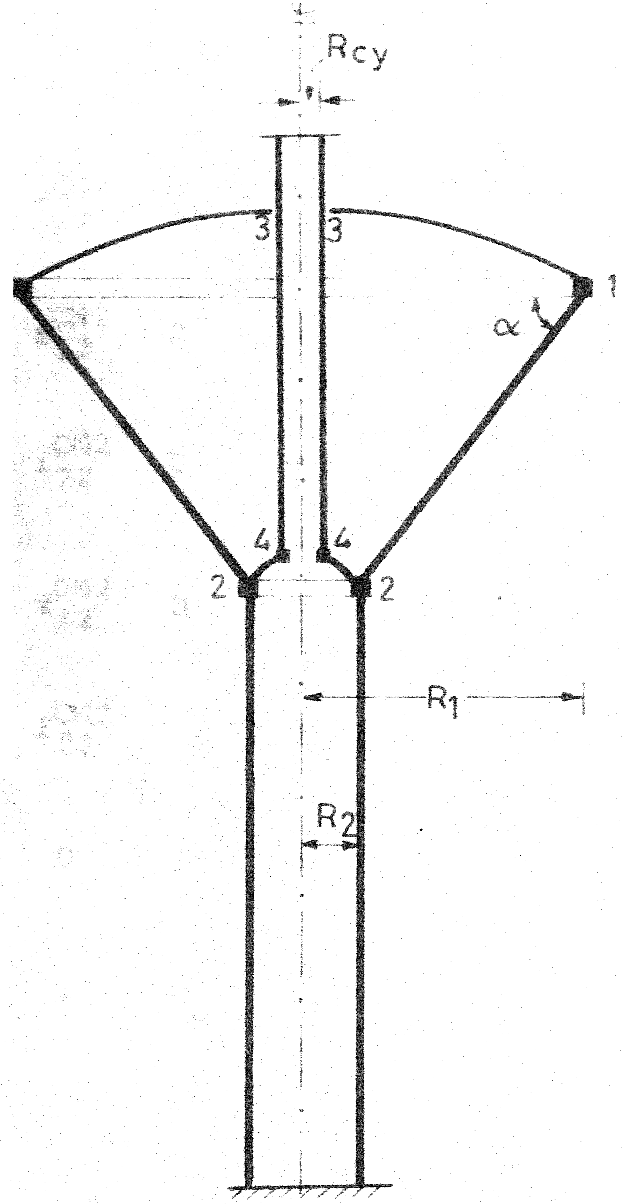


Fig. 4·1 Details of Type II water tower

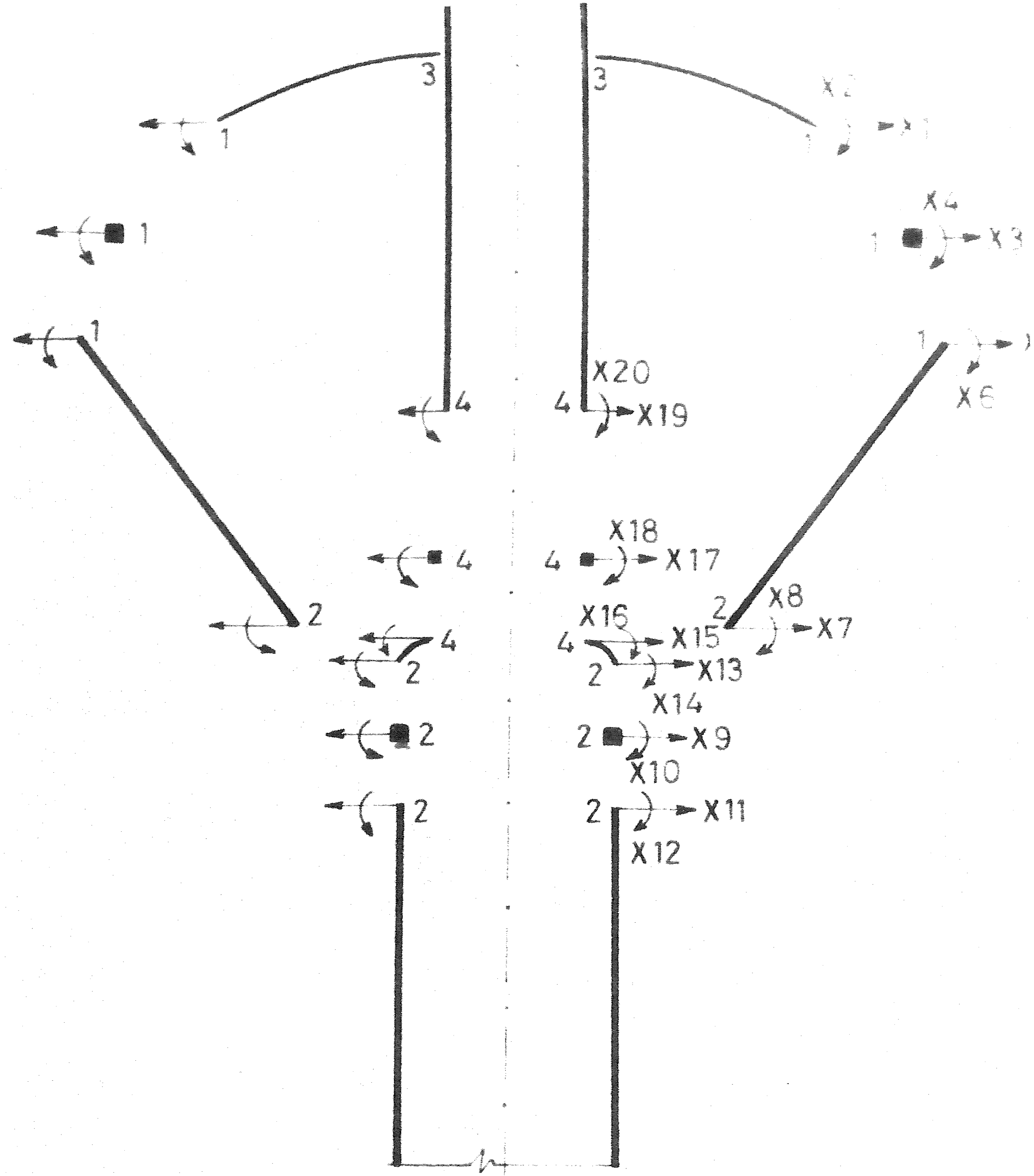


Fig 4·2 The corrective loadings at each junction (positive as

$$\begin{bmatrix} f_{11}^{CN2} & f_{12}^{CN2} & -f_{11}^{B2} & -f_{12}^{B2} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{21}^{CN2} & f_{22}^{CN2} & -f_{21}^{B2} & -f_{22}^{B2} & 0 & 0 & 0 & 0 & 0 & 0 \\ f_{11}^{CN2} & f_{12}^{CN2} & 0 & 0 & -f_{11}^{S2} & -f_{12}^{S2} & 0 & 0 & 0 & 0 \\ f_{21}^{CN2} & f_{22}^{CN2} & 0 & 0 & -f_{21}^{S2} & -f_{22}^{S2} & 0 & 0 & 0 & 0 \\ f_{11}^{CN2} & f_{12}^{CN2} & 0 & 0 & 0 & 0 & -f_{11}^{BD2} & -f_{12}^{BD2} & -cf_{11}^{BD2} & -cf_{12}^{BD2} \\ f_{21}^{CN2} & f_{22}^{CN2} & 0 & 0 & 0 & 0 & -f_{21}^{BD2} & -f_{22}^{BD2} & -cf_{21}^{BD2} & -cf_{22}^{BD2} \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X7 \\ X8 \\ X9 \\ X10 \\ X11 \\ X12 \\ X13 \\ X14 \\ X15 \\ X16 \end{bmatrix} = \begin{bmatrix} \delta_P^{B2} - \delta_P^{CN2} \\ \beta_P^{B2} - \beta_P^{CN2} \\ \delta_P^{S2} - \delta_P^{CN2} \\ \beta_P^{S2} - \beta_P^{CN2} \\ \delta_P^{BD2} - \delta_P^{CN2} \\ \beta_P^{BD2} - \beta_P^{CN2} \\ 0 \\ 0 \end{bmatrix} \tag{4.1}$$

where δ_p^{BD2} and β_p^{BD2} are the displacement and rotation of bottom dome at junction 2 due to primary loadings. f_{11}^{BD2}, f_{12}^{BD2}, f_{21}^{BD2} and f_{22}^{BD2} are the flexibility coefficients which represent displacements and rotations of bottom dome at junction 2 due to unit edge loadings on the dome at that junction itself, and cf_{11}^{BD2}, cf_{12}^{BD2}, cf_{21}^{BD2} and cf_{22}^{BD2} are the coefficients which give displacements and rotations of bottom dome at junction 2 due to unit edge-loadings at the other end of the dome.

Other terms have already been defined in Section 3.3.1.2.

Similarly the compatibility and equilibrium conditions for junction 4 will give a set of linear algebraic equations which can be expressed by Eq. (4.2).

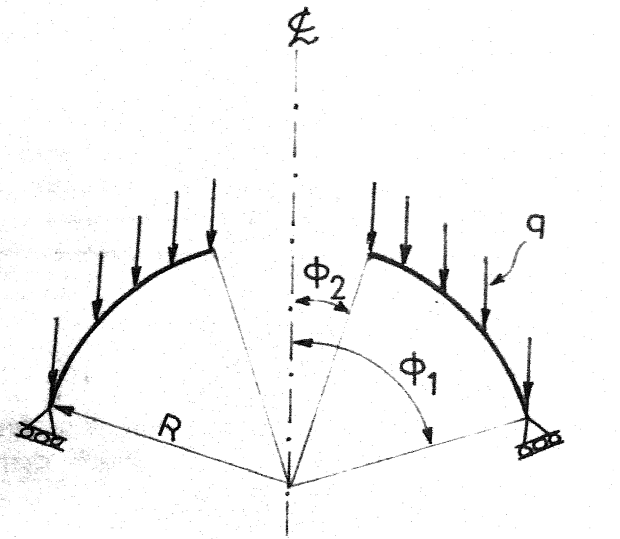
Now Eqs. (3.16), (4.1) and (4.2) are combined together to form a single set of twenty linear algebraic equations in twenty unknowns to arrive at the solution. The various flexibility coefficients and the deformations due to primary loadings, required for the solution, can be obtained from Tables 3.4 through 3.16 and 4.1.

The procedure given in Section 3.3.2 is adopted to carry out the elastic analysis for finding the design forces in the supporting shaft.

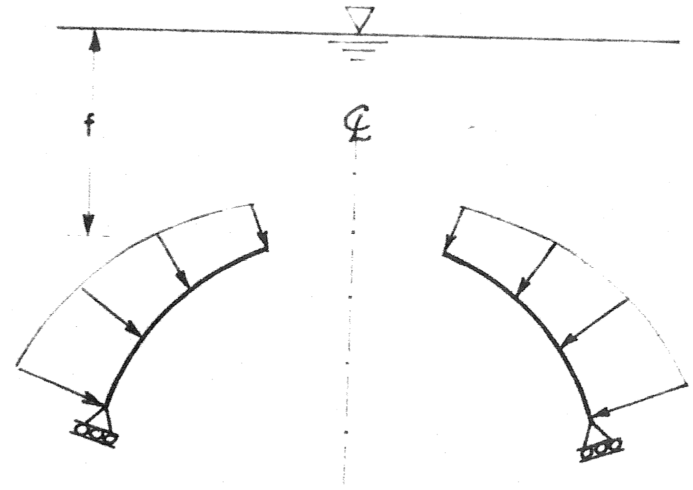
4.3 Limit Analysis

The limit analysis of conical tank discussed in Section 3.5.2 will be valid for this type of towers as well.

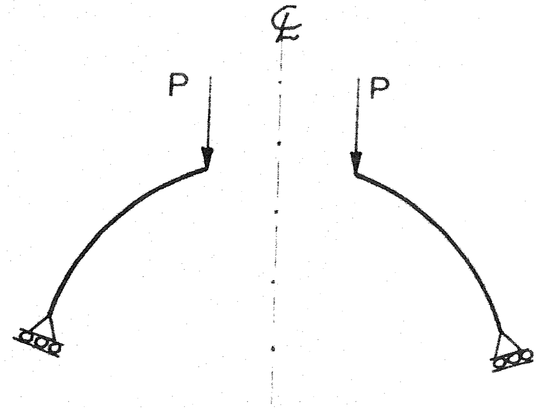
$$
\begin{bmatrix}
-cf_{11}^{BD4} & -cf_{12}^{BD4} & -f_{11}^{BD4} & -f_{12}^{BD4} & 0 & 0 & f_{11}^{CY4} & f_{12}^{CY4} \\
-cf_{21}^{BD4} & -cf_{22}^{BD4} & -f_{21}^{BD4} & -f_{22}^{BD4} & 0 & 0 & f_{21}^{CY4} & f_{22}^{CY4} \\
0 & 0 & 0 & 0 & -f_{11}^{B4} & -f_{12}^{B4} & f_{11}^{CY4} & f_{12}^{CY4} \\
0 & 0 & 0 & 0 & -f_{21}^{B4} & -f_{22}^{B4} & f_{21}^{CY4} & f_{22}^{CY4} \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X13 \\ X14 \\ X15 \\ X16 \\ X17 \\ X18 \\ X19 \\ X20
\end{bmatrix}
=
\begin{bmatrix}
\delta_P^{BD4} - \delta_P^{CY4} \\
\beta_P^{BD4} - \beta_P^{CY4} \\
\delta_P^{B2} - \delta_P^{CY4} \\
\beta_P^{B2} - \beta_P^{CY4} \\
0 \\
0
\end{bmatrix}
\qquad (4.2)
$$



(a) Dead weight loading



(b) Hydrostatic pressure loading



(c) Equally distributed load 'P' per unit length along the edge

Fig. 4·3 Primary loadings on bottom dome

Table 4.1 Membrane solutions for open spherical shell - primary loadings

Loading condition	Stress resultants and deformations	Algebraic expressions for stress resultants and deformations
1	2	3
Dead weight Refer Figure 4.3(a)	N_ϕ	$-Rq(\cos\phi_2 - \cos\phi)/\sin^2\phi$
	N_θ	$-Rq[\cos\phi - (\cos\phi_2 - \cos\phi)/\sin^2\phi]$
	δ	$R^2q[-\cos\phi + (1+\mu)(\cos\phi_2 - \cos\phi)/\sin^2\phi](\sin\phi)/Et$
	β	$-Rq(2+\mu)(\sin\phi)/Et$
Hydrostatic pressure loading. Refer Figure 4.3(b)	N_ϕ	$-\frac{\rho R^2}{6}[3(1+\frac{f}{R})(1-\frac{\sin^2\phi_2}{\sin^2\phi}) - 2\frac{\cos^3\phi_2 - \cos^3\phi}{\sin^2\phi}]$
	N_θ	$-\frac{\rho R^2}{6}[3(1+\frac{f}{R})(1+\frac{\sin^2\phi_2}{\sin^2\phi}) + \frac{2(2\cos^3\phi + \cos^3\phi_2) - 6\cos\phi}{\sin^2\phi}]$

Contd...

Table 4.1 (continued)

1	2	3
	δ	$- \frac{\rho R^3}{6Et} \{3(1 + \frac{f}{R})[1 - \mu + (1 + \mu)(\frac{\sin^2\phi_2}{\sin^2\phi})] - 6 \cos\phi + 2(1 + \mu) \frac{\cos^3\phi_2 - \cos^3\phi}{\sin^2\phi}\} \sin\phi$
	β	$\rho R^2 \sin\phi/Et$
Equally distributed loading along the edge. Refer Figure 4.3(c)	N_ϕ	$- P \sin\phi_2/\sin^2\phi$
	N_θ	$P \sin\phi_2/\sin^2\phi$
	δ	$PR(1 + \mu) \sin\phi_2/(Et \sin\phi)$
	β	0

where q = is the weight of shell per unit of its surface area.

The possibility of collapse of the bottom dome prior to the conical tank is also rare as it is primarily subjected to membrane compression. The limit analysis of the supporting shaft given in Section 3.5.3 will be applicable here also.

4.4 Optimization

4.4.1 Design variables

In addition to the variables considered for the Type I tower, two more design variables, one for the thickness, t_{bd}, and the another for the percentage of reinforcement, p_{rbd}, for the bottom dome are considered in the optimal design of Type II water towers. Thus, the total number of design variables become twelve for this case.

4.4.2 Objective function

The objective function of such a tower includes the cost of concrete, reinforcement and formwork for the bottom dome in addition to the costs considered for finding the objective function in case of Type I towers. Thus, the objective function can be written in the form

$$F = F' + 2\pi R_{bd}^{2}(\cos\phi_{2bd} - \cos\phi_{1bd})(t_{bd}C_c + \frac{2 \times p_{rbd}}{100} t_{bd}\,\rho_s\, C_s + 2C_f) \qquad (4.3)$$

where F' is the value of objective function given by Eq. (3.39) for a tower of Type I,

R_{bd} = radius of the bottom dome,

t_{bd} = thickness of the bottom dome,

p_{rbd} = percentage of reinforcement in the bottom dome for both circumferential and meridional direction, and

ϕ_{2bd} and ϕ_{1bd} are corresponding values of ϕ_2 and ϕ_1 (Figure 3.6), for the bottom dome.

4.4.3 Constraints

The first thirty constraints for this type of tower will be essentially same as those considered for the Type I towers. In addition to these, the constraints governing the design of the bottom dome are needed to be incorporated. Serviceability limit state of cracking demands that the flexural tensile stresses in concrete as well as in steel should remain within prescribed values as stated in Section 3.6.4. Thus, if M_{bdc} and M_{bds} are the maximum bending moment carrying capacities of unit width section of the bottom dome as per the limiting stresses for concrete and steel respectively, then the constraints thirty-one and thirty-two can be expressed in the form

$$\frac{M_{bda}}{M_{bdc}} - 1 \leq 0 , \qquad (4.4)$$

and

$$\frac{M_{bda}}{M_{bds}} - 1 \leq 0 \qquad (4.5)$$

where M_{bda} = actual maximum bending moment (maximum of M_{ϕ} and M_{θ}) per unit width of the bottom dome, at service loads.

Similarly for safety, the limit state of strength requires that the ultimate moment of resistance 'M_{urbd}' per unit width of bottom dome should be greater than or equal to the actual value of maximum ultimate moment 'M_{uabd}'. Thus, the thirty-third constraint can be written as

$$\frac{M_{uabd}}{M_{urbd}} - 1 \leq 0 \qquad (4.6)$$

In addition to these, two more constraints, regarding the minimum thickness and reinforcement for the dome can be expressed as

$$\frac{t_{min}}{t_{bd}} - 1 \leq 0 \qquad (4.7)$$

$$\frac{p_{r\ min}}{p_{bd}} - 1 \leq 0 \qquad (4.8)$$

Thus, the total number of constraints, for this case, comes out to be thirty-five.

4.5 Parametric Study

For water towers of Type II, optimal limit state design has been carried out for the following cases:

(1) capacity in m^3: 500, 750 and 1000;

(2) staging height in m: 15, 20 and 25; and

(3) basic wind pressure in kN/m^2: 1.0, 1.5 and 2.0.

A free board of about 200 mm has been provided in all the cases. The suitability of these designs also, obtained for different wind zones, is established for the various seismic zones of India.

Table 4.2 Optimal designs of Type II towers with 500 m^3 capacity tank

L (m)	Wind zone p_b (kN/m^2)	Optimal design variables						Optimal configuration parameters				Optimal cost (Rs)	Applicable to seismic zones
		t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	p_{rbd} t_{bd}	R_2 (m)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
25	2.0	174 150	1.436 0.354	0.980 1.101	0.632 1.717	0.25 0.25	0.50 150	2.0	3.5	1.007	9.36	241302	I-V
	1.5	170 153	1.443 0.285	1.029 0.617	0.654 1.265	0.25 0.25	0.50 150	2.0	3.5	1.007	9.36	233813	I-V
	1.0	174 153	1.417 0.284	1.011 0.630	0.679 1.227	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	223816	I-V
20	2.0	168 154	1.445 0.351	1.104 0.833	0.699 1.670	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	209878	I-V
	1.5	168 156	1.454 0.284	1.074 0.541	0.679 1.156	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	205791	I-IV
	1.0	171 150	1.423 0.256	1.038 0.366	0.708 0.697	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	201087	I-IV
15	2.0	173 156	1.380 0.255	1.036 0.467	0.675 0.735	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	187358	I-V
	1.5	175 150	1.363 0.293	1.050 0.320	0.660 0.617	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	185923	I-IV
	1.0	169 153	1.416 0.264	1.064 0.285	0.674 0.323	0.25 0.25	0.50 150	1.75	4.0	1.002	9.75	183565	I-III

Table 4.3 Optimal designs of Type II towers with 750 m^3 capacity tank

L (m)	Wind zone p_b (kN/m^2)	Optimal design variables						Optimal configuration parameters				Optimal cost (Rs)	Applicable to seismic zones
		t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	p_{rbd} t_{bd}	R_2 (m)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
25	2.0	219 199	1.442 0.367	1.041 0.889	0.686 1.773	0.25 0.25	0.50 150	2.0	4.0	1.001	11.12	329465	I–V
	1.5	221 208	1.434 0.268	1.064 0.593	0.689 1.157	0.25 0.25	0.50 150	2.0	4.0	1.001	11.12	323063	I–V
	1.0	223 214	1.459 0.258	1.059 0.585	0.663 1.324	0.25 0.25	0.50 150	1.75	4.5	1.020	11.70	314087	I–IV
20	2.0	220 198	1.406 0.279	1.072 0.510	0.685 1.196	0.25 0.25	0.50 150	2.0	4.0	1.001	11.12	296154	I–V
	1.5	221 231	1.429 0.281	1.067 0.544	0.670 0.925	0.25 0.25	0.50 150	1.75	4.5	1.020	11.70	293002	I–IV
	1.0	219 218	1.425 0.274	1.077 0.306	0.698 0.684	0.25 0.25	0.50 150	1.75	4.5	1.020	11.70	284658	I–III
15	2.0	219 226	1.436 0.271	1.097 0.388	0.697 0.741	0.25 0.25	0.50 150	1.75	4.5	1.020	11.70	268418	I–IV
	1.5	220 222	1.428 0.272	1.113 0.292	0.678 0.501	0.25 0.25	0.50 150	1.75	4.5	1.020	11.70	266072	I–IV
	1.0	219 206	1.425 0.292	1.064 0.298	0.705 0.509	0.25 0.25	0.50 150	1.75	4.5	1.020	11.70	263279	I–III

Table 4.4 Optimal designs of Type II towers with 1000 m^3 capacity tank

L (m)	Wind zone p_b (kN/m^2)	Optimal design variables t_c / t_s (mm)	p_{rc1} / p_{rs1}	p_{rc2} / p_{rs2}	p_{rc3} / p_{rs3}	p_{rc4} / p_{rcy}	p_{rbd} / t_{bd}	Optimal configuration parameters R_2 (m)	R_1/R_2	α (rad)	$\bar{L}$ (m)	Optimal cost (Rs)	Applicab[le] to seism[ic] zones
25	2.0	264 249	1.451 0.435	1.049 1.131	0.666 1.796	0.25 0.25	0.50 150	2.0	4.0	1.111	13.52	419668	I-V
	1.5	265 249	1.439 0.255	1.031 0.721	0.653 1.306	0.25 0.25	0.50 150	2.0	4.0	1.111	13.52	406106	I-V
	1.0	267 275	1.414 0.269	1.069 0.515	0.697 1.006	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	396643	I-IV
20	2.0	262 285	1.433 0.315	1.121 0.680	0.740 1.350	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	379031	I-V
	1.5	262 273	1.432 0.261	1.194 0.455	0.711 1.155	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	372477	I-IV
	1.0	265 281	1.412 0.271	1.077 0.314	0.706 0.535	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	366792	I-III
15	2.0	263 285	1.423 0.290	1.112 0.347	0.708 0.704	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	345021	I-IV
	1.5	263 281	1.425 0.274	1.086 0.320	0.709 0.433	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	341720	I-III
	1.0	264 267	1.400 0.283	1.116 0.360	0.724 0.428	0.25 0.25	0.50 150	1.75	5.0	0.991	12.77	341291	I-II

Table 4.5 Optimal designs of Type II towers with 500 m^3 capacity tank for non-optimal shaft radii

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	Optimal design variables						Optimal configuration parameters			Optimal cost (Rs)	Applicable to seismic zones
			t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	p_{rbd} t_{bd} (mm)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
25	2.0	1.75	174 202	1.396 0.586	1.061 1.115	0.660 1.838	0.25 0.25	0.50 150	4.0	1.002	9.75	252668	I-V
	1.5	1.75	175 164	1.370 0.359	1.001 1.188	0.651 1.663	0.25 0.25	0.50 150	4.0	1.002	9.75	234962	I-V
	1.0	1.50	170 170	1.439 0.346	1.084 0.942	0.764 1.615	0.25 0.25	0.50 150	5.0	0.898	9.63	226569	I-V
		2.00	172 150	1.424 0.253	1.013 0.380	0.670 0.848	0.25 0.25	0.50 150	3.5	1.007	9.36	228066	I-IV
20	2.0	1.50	170 192	1.434 0.437	1.088 1.162	0.761 1.815	0.25 0.25	0.50 150	5.0	0.898	9.63	217662	I-V
		2.0	174 150	1.407 0.288	0.999 0.580	0.666 0.969	0.25 0.25	0.50 150	3.5	1.007	9.36	212771	I-IV
	1.5	1.50	174 166	1.416 0.335	1.066 0.971	0.723 1.692	0.25 0.25	0.50 150	5.0	0.898	9.63	209409	I-IV
		2.0	173 153	1.419 0.253	0.991 0.398	0.641 0.643	0.25 0.25	0.50 150	3.5	1.007	9.36	210064	I-IV

Contd...

Table 4.5 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	1.0	1.50	170	1.439	1.088	0.743	0.25	0.50	5.0	0.898	9.63	203188	I-IV
			169	0.260	0.509	1.024	0.25	150					
		2.0	170	1.442	1.073	0.660	0.25	0.50	3.5	1.007	9.36	206818	I-III
			150	0.263	0.292	0.364	0.25	150					
15	2.0	1.50	170	1.457	1.113	0.757	0.25	0.50	5.0	0.898	9.63	190197	I-V
			171	0.265	0.600	1.172	0.25	150					
		2.0	169	1.446	1.029	0.668	0.25	0.50	3.5	1.007	9.36	190507	I-IV
			153	0.271	0.321	0.445	0.25	150					
	1.5	1.50	170	1.450	1.120	0.740	0.25	0.50	5.0	0.898	9.63	190680	I-IV
			187	0.317	0.409	0.627	0.25	150					
		2.0	169	1.447	1.027	0.683	0.25	0.50	3.5	1.007	9.36	189531	I-IV
			153	0.285	0.307	0.313	0.25	150					
	1.0	1.50	171	1.416	1.116	0.726	0.25	0.50	5.0	0.898	9.63	185641	I-IV
			170	0.279	0.312	0.464	0.25	150					
		2.0	170	1.425	1.069	0.636	0.25	0.50	3.5	1.007	9.36	189545	I-III
			154	0.272	0.275	0.278	0.25	150					

'able 4.6 Optimal designs of Type II towers with 750 m^3 capacity tank for non-optimal shaft radii

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	Optimal design variables						Optimal configuration parameters			Optimal cost (Rs)	Applicable to seismic zones
			t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	p_{rbd} t_{bd} (mm)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
25	2.0	1.75	222 246	1.393 0.620	1.091 1.380	0.684 1.912	0.25 0.25	0.50 150	4.5	1.020	11.70	341809	I-V
		2.25	226 195	1.425 0.275	1.048 0.606	0.642 1.202	0.25 0.25	0.50 150	3.5	1.027	10.87	333406	I-V
	1.5	1.75	219 224	1.438 0.408	1.075 0.922	0.696 1.709	0.25 0.25	0.50 150	4.5	1.020	11.70	323219	I-IV
		2.25	226 187	1.431 0.271	0.995 0.404	0.669 0.967	0.25 0.25	0.50 150	3.5	1.027	10.87	325580	I-IV
	1.0	1.5	224 239	1.432 0.437	1.061 1.053	0.673 1.877	0.25 0.35	0.50 150	5.0	1.082	12.76	317320	I-IV
		2.0	226 203	1.418 0.270	1.069 0.337	0.655 0.788	0.25 0.25	0.50 150	4.0	1.001	11.12	318214	I-IV
20	2.0	1.75	223 226	1.388 0.311	1.084 0.877	0.687 1.370	0.25 0.25	0.50 150	4.5	1.020	11.70	298070	I-V
		2.25	222 185	1.422 0.274	1.022 0.350	0.650 0.807	0.25 0.25	0.50 150	3.5	1.027	10.87	297555	I-IV

Contd...

Table 4.6 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	1.5	1.5	220 247	1.429 0.281	1.100 0.544	0.706 0.925	0.25 0.35	0.50 150	5.0	1.082	12.76	295836	I-IV
		2.0	223 203	1.415 0.277	1.040 0.339	0.655 0.704	0.25 0.25	0.50 150	4.0	1.001	11.12	293482	I-III
	1.0	1.5	224 237	1.386 0.310	1.077 0.615	0.687 1.165	0.25 0.35	0.50 150	5.0	1.082	12.76	285440	I-III
		2.0	227 195	1.364 0.291	1.030 0.328	0.645 0.440	0.25 0.25	0.50 150	4.0	1.001	11.12	291965	I-III
15	2.0	1.5	222 262	1.406 0.315	1.081 0.878	0.663 1.100	0.25 0.35	0.50 150	5.0	1.082	12.76	271211	I-V
		2.0	226 177	1.414 0.277	1.060 0.897	0.655 1.347	0.25 0.25	0.50 150	4.0	1.001	11.12	276965	I-IV
	1.5	1.5	220 253	1.411 0.364	1.095 0.697	0.674 0.831	0.25 0.35	0.50 150	5.0	1.082	12.76	266506	I-IV
		2.0	222 198	1.423 0.270	1.061 0.303	0.674 0.306	0.25 0.25	0.50 150	4.0	1.001	11.12	268957	I-III
	1.0	1.5	224 240	1.413 0.310	1.096 0.345	0.658 0.636	0.25 0.35	0.50 150	5.0	1.082	12.76	263424	I-III
		2.0	220 188	1.422 0.266	1.047 0.305	0.691 0.306	0.25 0.25	0.50 150	4.0	1.001	11.12	265500	I-II

Table 4.7 Optimal designs of Type II towers with 1000 m^3 capacity tank for non-optimal shaft radii

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	Optimal design variables						Optimal configuration parameters			Optimal cost (Rs)	Applicabl[e] to seismi[c] zones
			t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	p_{rbd} t_{bd} (mm)	R_1/R_2	α (rad)	$\bar{L}$ (m)		
1	2	3	4	5	6	7	8	9	10	11	12	13	14
25	2.0	1.75	266 292	1.393 0.579	1.109 1.283	0.718 1.908	0.25 0.25	0.50 150	5.0	0.991	12.77	426241	I-V
		2.25	274 245	1.418 0.290	1.023 0.687	0.606 1.287	0.25 0.25	0.50 150	3.5	1.133	13.28	424264	I-V
	1.5	1.75	265 277	1.399 0.349	1.080 0.889	0.711 1.574	0.25 0.25	0.50 150	5.0	0.991	12.77	407626	I-V
		2.25	267 230	1.457 0.281	1.043 0.573	0.647 1.056	0.25 0.25	0.50 150	3.5	1.133	13.28	411758	I-IV
	1.0	1.5	289 313	1.310 0.398	1.017 0.969	0.637 1.522	0.25 0.35	0.50 150	5.5	1.068	14.02	411397	I-IV
		2.0	271 260	1.415 0.261	1.033 0.386	0.647 0.732	0.25 0.25	0.50 150	4.0	1.111	13.52	401780	I-IV
20	2.0	1.5	284 329	1.313 0.626	1.031 1.400	0.665 1.855	0.25 0.35	0.50 150	5.5	1.068	14.02	398087	I-V
		2.0	266 261	1.414 0.354	1.035 0.484	0.658 1.059	0.25 0.25	0.50 150	4.0	1.111	13.52	379261	I-V

Contd...

Table 4.7 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
	1.5	1.5	288 318	1.298 0.364	1.023 0.963	0.655 1.591	0.25 0.35	0.50 150	5.5	1.068	14.02	386941	I-IV
		2.0	267 246	1.402 0.258	1.059 0.336	0.655 0.970	0.25 0.25	0.50 150	4.0	1.111	13.52	372546	I-IV
	1.0	1.5	283 314	1.306 0.251	1.033 0.492	0.660 1.034	0.25 0.35	0.50 150	5.5	1.068	14.02	372711	I-III
		2.0	266 256	1.417 0.268	1.050 0.308	0.664 0.412	0.25 0.25	0.50 150	4.0	1.111	13.52	368328	I-III
15	2.0	1.5	288 335	1.298 0.299	0.990 0.739	0.642 1.052	0.25 0.35	0.50 150	5.5	1.068	14.02	357715	I-V
		2.0	270 253	1.413 0.272	1.015 0.285	0.642 0.653	0.25 0.25	0.50 150	4.0	1.111	13.52	347562	I-IV
	1.5	1.5	283 325	1.304 0.259	1.041 0.463	0.658 0.885	0.25 0.35	0.50 150	5.5	1.068	14.02	350929	I-III
		2.0	265 249	1.413 0.275	1.033 0.335	0.659 0.406	0.25 0.25	0.50 150	4.0	1.111	13.52	343103	I-III
	1.0	1.5	288 317	1.279 0.274	1.035 0.340	0.663 0.513	0.25 0.35	0.50 150	5.5	1.068	14.02	349023	I-II
		2.0	268 247	1.426 0.252	1.019 0.267	0.651 0.280	0.25 0.25	0.50 150	4.0	1.111	13.52	342805	I-II

Table 4.8 Effect of cost ratios on optimal configuration

Capacity of tank = 750 m^3, L = 25 m, p_b = 1.0 kN/m^2

S. No.	CR1	CR2	R_2 (m)	R_1/R_2	Optimal cost (Rs)	Optimal configuration
1	1.25	0.10	1.5	4.5	332490	
				5.0	317320	
				5.5	321577	R_2 = 1.75 m,
			1.75	4.0	319917	R_1/R_2 = 4.5 or
				4.5	314087	α = 1.020 rad
				5.0	322593	
			2.0	3.5	321087	
				4.0	318214	
				4.5	330318	
2	2.0	0.10	1.5	4.5	411239	
				5.0	391400	
				5.5	397272	R_2 = 1.75 m,
			1.75	4.0	396683	R_1/R_2 = 4.5 or
				4.5	390498	α = 1.020 rad
				5.0	401500	
			2.0	3.5	404041	
				4.0	395256	
				4.5	416977	
3	1.25	0.15	1.5	4.5	382058	
				5.0	362739	
				5.5	364515	R_2 = 1.75 m,
			1.75	4.0	370597	R_1/R_2 = 4.5 or
				4.5	357754	α = 1.020 rad
				5.0	373066	
			2.0	3.5	370837	
				4.0	362330	
				4.5	384219	

Table 4.9 Effect of cost ratios on optimal design variables

Capacity of tank = 750 m^3, L = 25 m, p_b = 1.0 kN/m^2

S. No.	R_2 (m)	$\frac{R_1}{R_2}$	CR1	CR2	Optimal design variables						Optimal cost
					t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	p_{rbd} t_{bd} (mm)	(Rs)
1	1.75	4.5	1.25	0.10	223 214	1.459 0.258	1.060 0.585	0.663 1.324	0.25 0.25	0.50 150	314087
2	1.75	4.5	1.25	0.15	219 210	1.450 0.292	1.083 0.674	0.708 1.236	0.25 0.25	0.50 150	357754
3	1.75	4.5	2.00	0.10	222 217	1.405 0.280	1.070 0.658	0.670 1.158	0.25 0.25	0.50 150	390498
4	1.75	4.5	0.75	0.10	219 213	1.454 0.262	1.085 0.742	0.681 1.115	0.25 0.25	0.50 150	260065

Table 4.10 Comparative optimal cost study of Type I and Type II water towers

$L = 20$ m, $p_b = 2.0$ kN/m^2

Tank capacity (m^3)	Optimal cost of Type I tower (Rs)	Optimal cost of Type II tower (Rs)	Difference in costs of two towers (Rs)
100	99196	97197	1999
200	132777	130799	1978
300	164292	158762	5530
500	245902	209878	36024
750	347775	296154	51621
1000	456750	379031	77719

4.6 Results and Discussions

Similar to the case of Type I towers, for this type of towers also it has been observed that for optimal configuration the bending moment and shearing force in the reservoir portion reduce considerably and the value of maximum hoop tension becomes the minimum, in most of the cases.

The Tables 4.2 through 4.4 give optimal designs of Type II towers for various capacities, staging heights and wind and seismic zones. Whereas, Tables 4.5 through 4.7 give optimal designs for these cases when the radius of the supporting shaft is other than the optimal one but with a value in its neighbourhood.

From the results given in Tables 4.8 and 4.9 it can be observed that, for this type of towers also, the effect of various cost ratios on optimal configuration and design variables is quite similar to that already found for Type I towers. Thus, optimal configuration remains unaffected by any change in the cost of materials. However, a slight variation observed in the optimal values of some of the design variables is negligible for all practical purposes. Hence present optimal designs will remain optimal or almost optimal, in future, for different costs of materials.

The optimal value of α is found to lie between 0.991 to 1.111 radian for all the cases but for most of these it is between 0.991 to 1.02 radian. The optimal value of R_2 is found to be 1.75 m for most capacities, staging heights and wind and seismic zones.

From the Table 4.10 it is evident that for capacities upto 300 kl the difference between the costs of two types of towers viz. Type I and Type II is marginal (1 to 3.5%). However for capacities 500 kl and above the Type II towers are quite economical. Although the cost of the Type I towers for capacities upto 300 kl is slightly higher than the Type II towers, from the constructional point of view the former are simpler. Hence the preference of Type I towers over Type II upto 300 kl capacity is justifiable.

Discussion on relative values of seismic and wind forces as well as those regarding constraints which are active except the ones dealing with minimum thickness of tank wall and supporting shaft given in Section 3.9, is found to be valid for this type of towers also. In addition to these, the constraint which deals with the ultimate strength of a section of the conical shell at junction 2 is also found to be active in some cases of 1000 m^3 capacity tank.

4.7 Variable Thickness of the Conical Tank Wall

Although the analysis as well as the optimal designs of the water towers described in the foregoing are strictly applicable when the thicknesses of the various elements are uniform, still the thickness of the wall of the conical tank can be varied without causing any serious error in the values of the forces which govern the design of the tank. The location of the maximum hoop tension in the conical tank is invariably found to be between 0.3 $\bar{L}$ to 0.5 $\bar{L}$ from the junction 2, for all the cases. From 0.5 $\bar{L}$ to the junction 1,

the hoop tension decreases uniformly and becomes almost equal to zero at junction 1. Thus, the thickness of the conical tank wall can be decreased uniformly from the maximum, equal to that obtained in the optimal design, at 0.5 $\bar{L}$ to a minimum prescribed thickness, 125-150 mm, at the junction 1. The area of hoop reinforcement to be provided in each of the three regions will remain equal to that provided for the optimal design. This variation of the thickness will not lead to any error, particularly, in the values of hoop tension at locations which govern the design of the tank wall and also does not violate any of the governing design constraints. However, there may be a little change in the bending moments in the edge region of junction 1 which can be taken care by providing slightly thicker section for a short length near junction 1. Such a taper results in saving of a considerable volume of concrete, particularly, in case of 750 and 1000 m^3 capacity tanks.

4.8 Example

To illustrate the method of using the design tables for the type of the towers considered, a design example has been taken.

Problem: A conical water tank supported on cylindrical shaft is required to be designed for a capacity of 750 m^3, staging height 20 m and wind and seismic zones II and III respectively.

The Table 4.3 gives the optimal designs for 750 m^3 capacity tanks. From this table the various design variables

can be found for the given staging height and wind and seismic zones. The optimal design values are as follows:

Radius of the supporting shaft 'R_2' = 1.75 m.

Value of R_1/R_2 = 4.5 (α = 1.020).

Length of the conical wall '$\bar{L}$' = 11.70 m.

Thickness of conical tank wall 't_c' = 221 mm.

Areas of hoop reinforcement in the three regions of the conical portion (Figure 3.15) in order are 3158, 2358 and 1480 mm^2/m.

Area of meriodional reinforcement in the conical portion (as per minimum requirement of 0.25%) = 552.5 mm^2/m.

Thickness of the supporting shaft 't_s' = 231 mm.

Areas of vertical reinforcement in the three regions of the supporting shaft (Figure 3.16) in order are 649, 1257 and 2136 mm^2/m.

Area of circumferential reinforcement for supporting shaft (as per minimum requirement of 0.25%) = 578 mm^2/m.

Thickness of the bottom dome 't_{bd}' = 150 mm

Area of reinforcement in meridional as well as in circumferential direction for the bottom dome is = 750 mm^2/m.

A 100 mm thick top dome with 250 mm^2/m reinforcement in both the directions and the inner cylinder of 125 mm thickness with 312.5 mm^2/m reinforcement in the two directions is found to be adequate, for all the cases.

Ring beams of 330 x 330 mm and 225 x 225 mm sizes with 0.5% hoop reinforcements will be adequate enough for junctions 1 and 2, and 4 respectively.

The variation of the thickness of the wall of the conical tank can be done as per the procedure described in Section 4.7. The thickness of the wall from junction 2 to 0.5 $\bar{L}$ (5.85 m) will remain equal to 221 mm itself. From 0.5 $\bar{L}$ (5.85 m) to the point of junction 1 it can be uniformly decreased to a minimum of 125-150 mm. The area of hoop reinforcement will essentially remain unchanged in all the three regions and will be equal to 3158, 2358 and 1480 mm^2/m respectively. However, for a short length near junction 1 a slightly thicker section with gradually increasing thickness should be provided.

CHAPTER 5

OPTIMAL DESIGN OF TYPE III WATER TOWERS

5.1 Design Considerations

Sometimes a reservoir is required to be provided with a partition for various reasons. In this chapter, water towers in which reservoirs having partitions are considered. The Figure 1.1(c) shows a tower of this type having a cylindrical partition which divides the reservoir in two portions. Water towers of this type are also considered for 500 to 1000 kl capacities.

Design forces for the reservoir portion are obtained by considering all the possible governing axisymmetric loading combinations such as, both the portions of the tank are full, only outer portion is full and when only inner portion is full. It has been observed that the bending forces in edge region of junction 2, in all the elements, are found to be more when only outer portion is full, whereas the governing design forces, particularly the hoop tension, for the cylindrical partition is obtained when only inner portion is full (Figure 5.3). In addition to these, all the design considerations which are discussed in the Sections 3.4 and 4.1 will be valid for this type of towers also. The minimum thickness of the partition wall is restricted to 125 mm.

5.2 Elastic Analysis

The force method of analysis described in Section 3.3.1 is used for elastic analysis for various axisymmetric loading combinations discussed in the previous section. The Figure 5.1 shows the details of the water tower and the Figure 5.2 shows the same, exploded into simple shell elements with corrective edge-loadings at different junctions.

At junctions 1 and 4, the compatibility of deformations and equilibrium conditions give relations which will be exactly the same as those given by Eqs. (3.16) and (4.2). Whereas, for junction 2 it will be slightly different from the one given for Type II towers. For this type of towers, equilibrium and compatibility conditions at junction 2 will yield an equation of the form given by Eq. (5.1) in which

f_{11}^{CY2}, f_{12}^{CY2}, f_{21}^{CY2} and f_{22}^{CY2} are the appropriate flexibility coefficients for the cylindrical partition,

and

δ_p^{CY2} and β_p^{CY2} represent displacement and rotation respectively, of the cylindrical partition due to primary loading, at junction 2.

The flexibility coefficients and deformations due to primary loading for cylindrical partition can be obtained from Tables 3.6 and 3.10. The results of the Table 3.10 should be used with proper sign. These results are to be used with same sign when hydrostatic pressure causes hoop compression and with opposite sign in case of hoop tension.

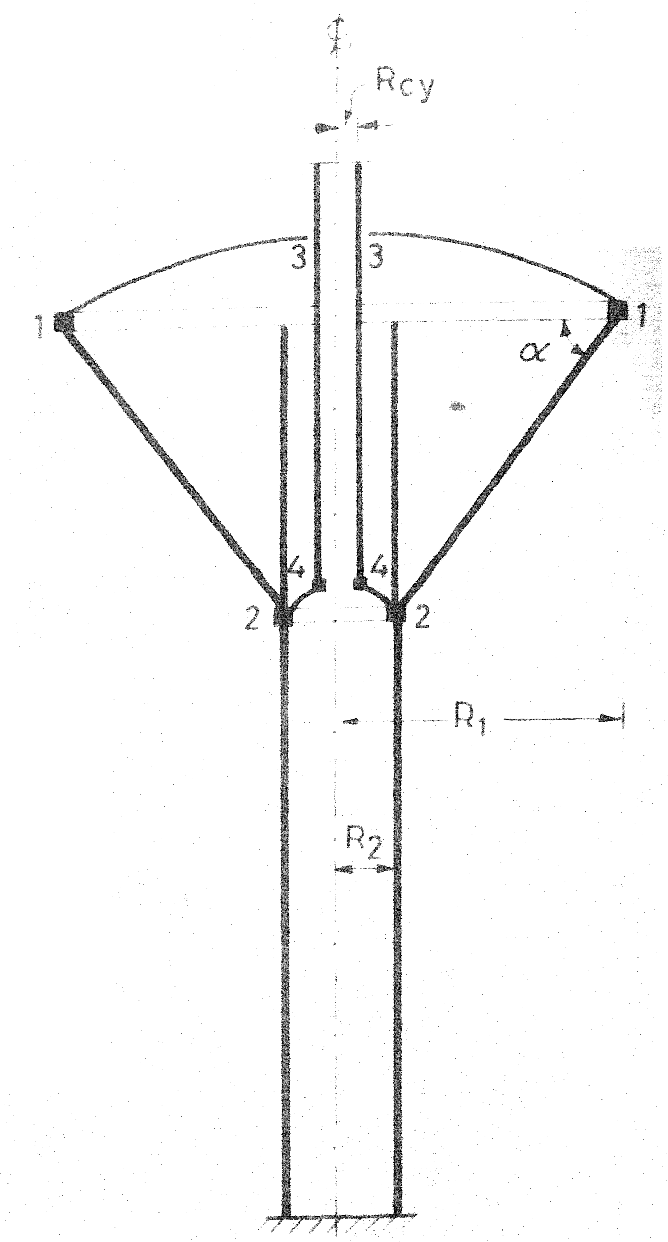


Fig.5·1 Details of Type III water tower

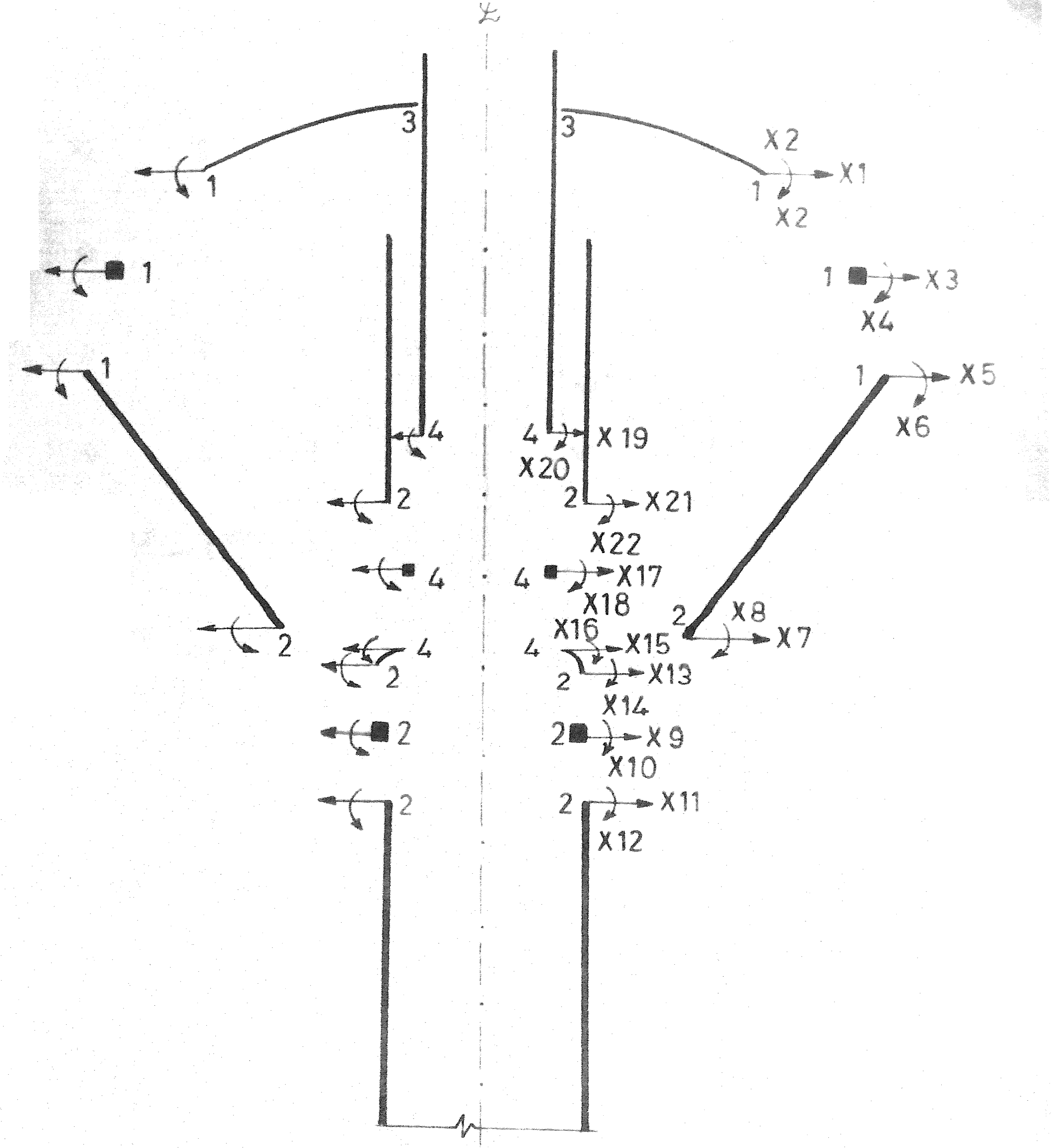


Fig.5·2 The corrective loadings at each junction (positive as shown)

$$
\begin{bmatrix}
f_{12}^{CN2} & -f_{11}^{B2} & -f_{12}^{B2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f_{22}^{CN2} & -f_{21}^{B2} & -f_{22}^{B2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f_{12}^{CN2} & 0 & 0 & -f_{11}^{S2} & -f_{12}^{S2} & 0 & 0 & 0 & 0 & 0 & 0 \\
f_{22}^{CN2} & 0 & 0 & -f_{21}^{S2} & -f_{22}^{S2} & 0 & 0 & 0 & 0 & 0 & 0 \\
f_{12}^{CN2} & 0 & 0 & 0 & 0 & -f_{11}^{BD2} & -f_{12}^{BD2} & -cf_{11}^{BD2} & -cf_{12}^{BD2} & 0 & 0 \\
f_{22}^{CN2} & 0 & 0 & 0 & 0 & -f_{21}^{BD2} & -f_{22}^{BD2} & -cf_{21}^{BD2} & -cf_{22}^{BD2} & 0 & 0 \\
f_{12}^{CN2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -f_{11}^{CY2} & -f_{12}^{CY2} \\
f_{22}^{CN2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -f_{21}^{CY2} & -f_{22}^{CY2} \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X7 \\ X8 \\ X9 \\ X10 \\ X11 \\ X12 \\ X13 \\ X14 \\ X15 \\ X16 \\ X21 \\ X22
\end{bmatrix}
=
\begin{bmatrix}
\delta_P^{B2} - \delta_P^{CN2} \\
\beta_P^{B2} - \beta_P^{CN2} \\
\delta_P^{S2} - \delta_P^{CN2} \\
\beta_P^{S2} - \beta_P^{CN2} \\
\delta_P^{BD2} - \delta_P^{CN2} \\
\beta_P^{BD2} - \beta_P^{CN2} \\
\delta_P^{CY2} - \delta_P^{CN2} \\
\beta_P^{CY2} - \beta_P^{CN2} \\
0 \\
0
\end{bmatrix}
\qquad (5.1)
$$

Thus the Eqs. (3.16), (4.2) and (5.1), when combined together, form a single set of twenty-two algebraic equations in twenty-two unknowns (X1-X22) which can be solved by matrix inversion. The various elements of the flexibility matrix and the components of the deformation vector due to primary loading can be obtained by using the results given in Tables 3.4 through 3.16 and 4.1.

For finding the design forces in the supporting shaft, the procedure given in Section 3.3.2 is adopted, for the elastic analysis.

5.3 Distribution and Arrangement of Reinforcements in the Cylindrical Partition

The Figure 5.3 shows general pattern of variation of stress resultants along x-direction for the partition. It can be seen that the value of hoop tension 'N_θ' is small for a very short length at the bottom of the partition and it shoots up to its maximum value at $\frac{x}{l} \simeq 0.1$ (where l is the length of the cylindrical partition) and then gradually decreases to zero value at its top. Curtailment of hoop reinforcement for such a short length at the bottom will not be of much use, therefore, a stepped distribution of reinforcement with points of curtailment at one-third and two-third heights of the partition has been adopted in all the cases. Similarly, the maximum bending moment 'M_x' occurs at the bottom and it decreases at exponential rate. Therefore, a minimum reinforcement of 0.25% will be adequate in x-direction except

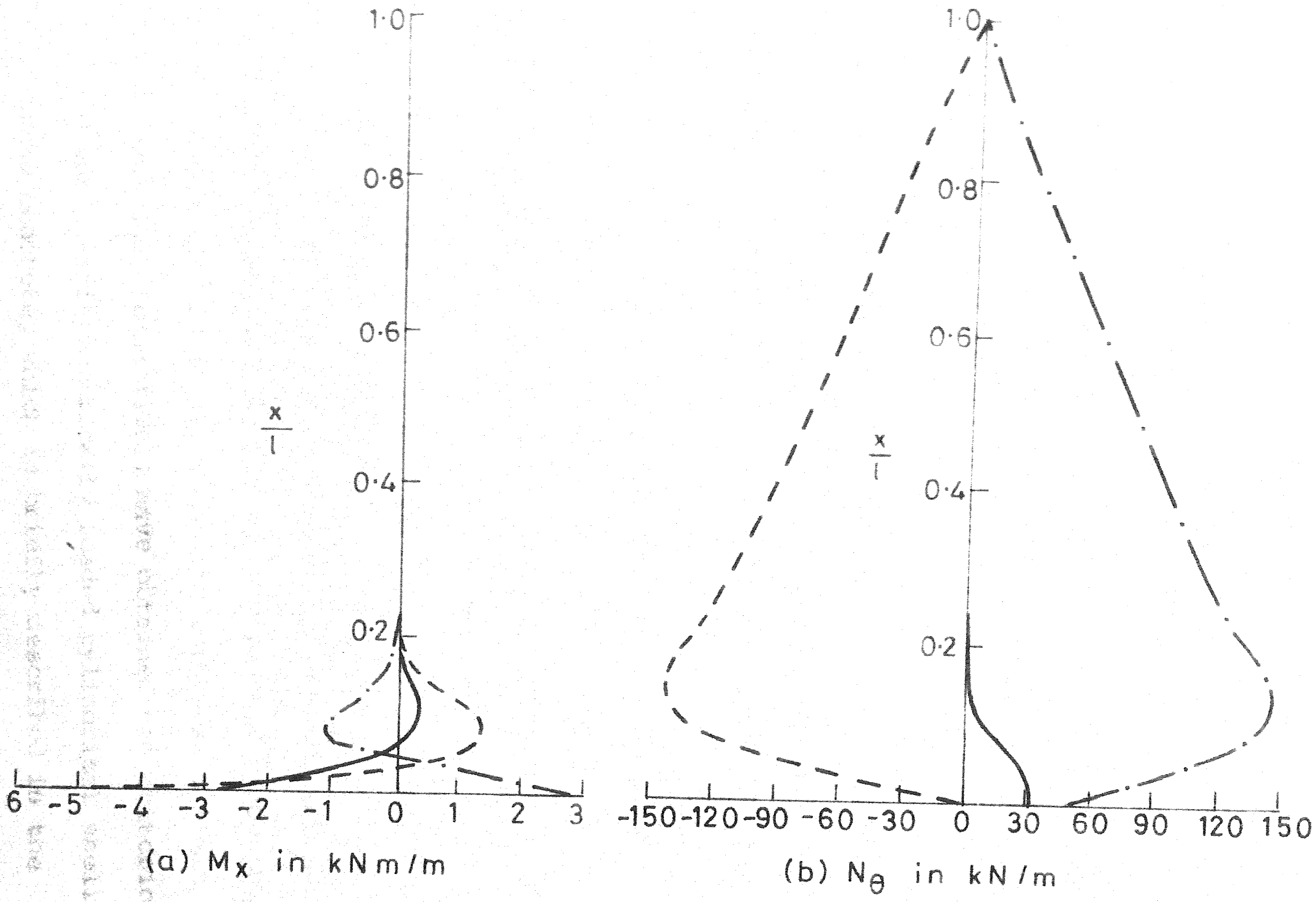


Fig. 5·3 Variation of stress resultants in x-direction for cylindrical partition

than 1 m in all the cases considered in the present study. Hence in finding the objective function, this length is taken as 1 m. The hoop reinforcement as well as the reinforcement in the vertical direction is provided on both the faces, 50% on each side, in a staggered fashion.

5.4 Limit Analysis

The limit analysis of conical tank and that of supporting shaft described in Sections 3.5.2 and 3.5.3 will be valid for this type of towers as well. In addition to these, there is a possibility of collapse of cylindrical partition also when only inner portion of the tank is full. In the limit analysis of structures, it is a common practice to get a lower bound on the load carrying capacity from a statically admissible stress field or to obtain an upper bound from a kinematically admissible mechanism. In case of reinforced concrete shells it is very much tedious to obtain an exact solution through lower bound approach except for some simple cases. However, in the case of cylindrical partition this can be applied without much of difficulty. For this, the solution commences from the equilibrium equation and a kinematically admissible mechanism for the stress field satisfying the equation of equilibrium is then found. Hence the solution so obtained is complete and gives the exact limit load. Sawczuk and Olszak (1961) have obtained the criterion of failure for an axisymmetrically loaded cylindrical shell of reinforced concrete, which is briefly described in the

5.4.1 Criterion of failure

For an axisymmetric shell there are, in general, four stress resultants; the bending moment M_x, the shear force Q_x and membrane forces N_x and N_θ. When Q_x is neglected in the condition of failure, the failure criterion is represented by two parabolic cylinders bounded by two parallel planes. If force N_x is small in comparison with N_θ and M_x, which is true for the case under consideration in the present study, the intersection of the cylinders with the plane $N_x = 0$, gives the simplified failure criterion in the M_x-N_θ plane. The Figure 5.4 shows the simplified failure criterion and the rule of instantaneous deformations. In this figure, χ_x and λ_θ denote the generalised strains associated with M_x and N_θ and M_o = ultimate moment of resistance per unit width, to resist positive moment; M_o' = ultimate moment of resistance per unit width, to resist negative moment; N_c = ultimate hoop compression capacity per unit width; and N_t = ultimate hoop tension capacity per unit width, all pertaining to any section of the shell under consideration. The moment and membrane force capacities can vary from section to section, but the profile of the failure criterion remains essentially the same.

The Figure 5.5 shows the forces acting on an element of the shell under consideration and the sign convention adopted. The equation of equilibrium for this case of cylindrical shell can be written as

$$\frac{d^2M_x}{.\,2} - \frac{N_\theta}{R} - p_x = 0 \qquad (5.2)$$

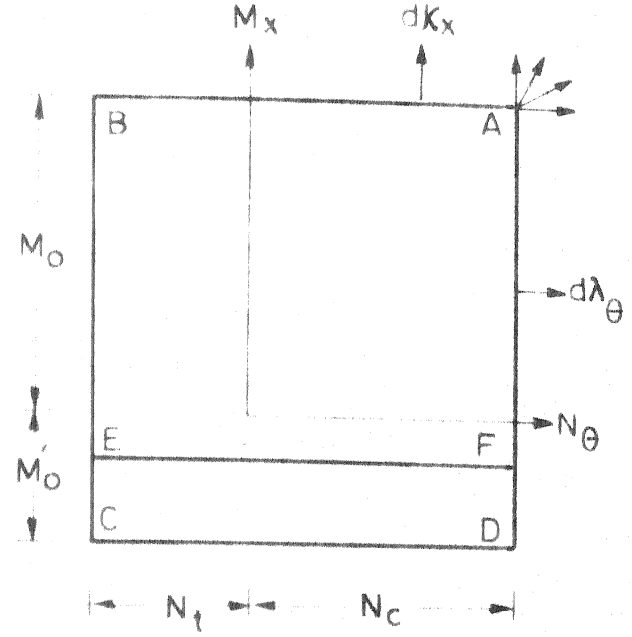


Fig. 5·4 Simplified failure criterion

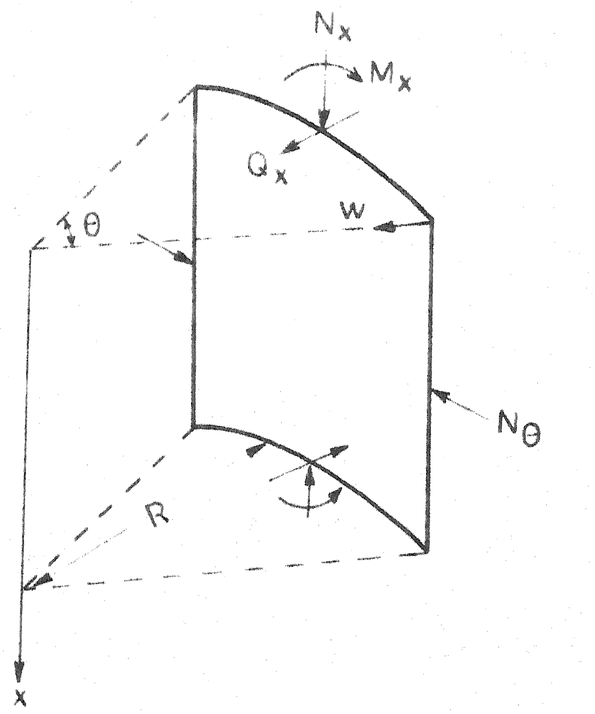
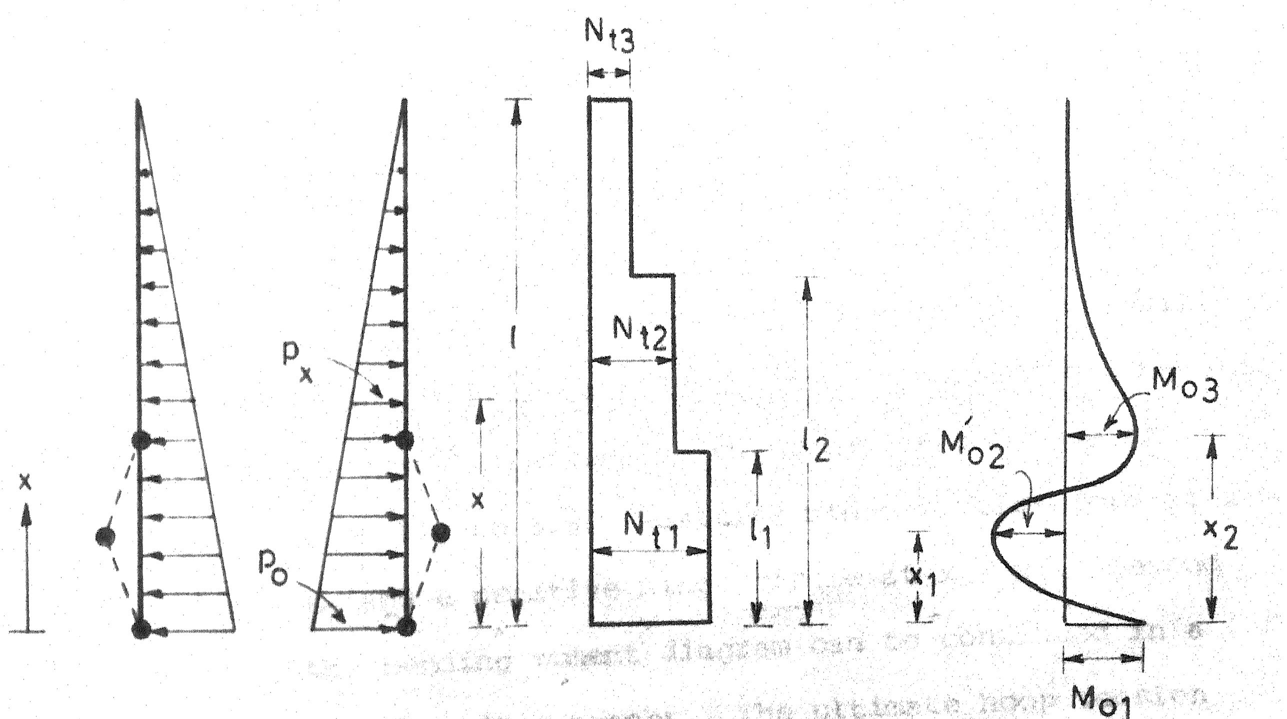


Fig. 5·5 Forces acting on element of shell and sign convent



where p_x = intensity of hydrostatic pressure,

R = radius of the cylindrical shell, and

w = radial displacement.

The generalised strain rates can be expressed in terms of w in the form $\dot{X}_x = - d^2\dot{w}/dx^2$ and $\dot{\lambda}_\theta = \dot{w}/R$. From the Figure 5.4 it can be seen that regimes AB and CD have to be rejected because along AB and CD, $\dot{w}$ is equal to zero and $d^2\dot{w}/dx^2$ is not equal to zero, which is impossible. Along BC and AD, $\dot{X}_x$ is equal to zero which leads to possible collapse mechanism of the form $\dot{w} = c_1x + c_2$. Therefore the collapse mode is always conical in form. The exact shape of the collapse mechanism depends on the shell parameters, and moment and hoop tension capacity distribution provided. Based on these, shells may be classified as short, medium and long.

5.4.2 Collapse mechanism

The size of the cylindrical partition considered in the current investigation is such that it will be always in the category of long cylinders for which the failure will be caused by partial collapse of the bottom portion of the shell, in all the cases. Details of this mode of collapse are given in the Figure 5.6. Positive hinge circles form at $x = 0$ and $x = x_2$ and a negative hinge circle at $x = x_1$. Beyond $x = x_2$, the bending moment diagram can be continued in a statically admissible manner. The ultimate hoop tension ... distribution conforms to the hoop reinforcement

distribution, described in Section 5.3, provided for crack control at service loads. The value of ultimate hoop tension capacity jumps from N_{t3} to N_{t2} at the second region boundary $x = l_2$ and from N_{t2} to N_{t1} at the first region boundary $x = l_1$, which is permissible; but the bending moment and shear force have to be continuous. Due to the hydrostatic loading the value of p_x will be equal to $p_o(1 - \frac{x}{l})$, where p_o is the collapse pressure. The equation of equilibrium can be written as

$$\frac{d^2M_x}{dx^2} - \frac{N_\theta}{R} - p_o(1 - \frac{x}{l}) = 0 \tag{5.3}$$

Integration of this equation, in the bottom two regions, results in four constants of integration. In addition to the moment and shear continuity conditions at the first region boundary $x = l_1$, the condition that $M_x = M_{01}$ at $x = 0$ and $\frac{dM_x}{dx} = 0$ at $x = x_2$ are used to evaluate these constants of integration. The value of these constants will depend upon the location of third hinge circle. For the type of hoop tension capacity distribution considered herein, the third hinge will usually be in the bottom region or in some cases it may be in the middle region. Depending upon the location of third and second hinge circles, three cases are considered as under:

Case I

When the third hinge circle is assumed to be in the

simplification, can be given by the following equations:

$$\frac{dM_x}{dx} = \frac{p_o}{2l}(2lx - x^2 - 2lx_2 + x_2^2) - \frac{N_{t1}}{R}(x - x_2) , \qquad (5.4)$$

and
$$M_x = \frac{p_o}{6l}(3lx^2 - x^3 - 6lx_2x + 3x_2^2x)$$

$$- \frac{N_{t1}}{2R}(x^2 - 2x_2x) + M_{01} \qquad (5.5)$$

In order to solve for the three unknowns p_o, x_1 and x_2, the following conditions are used:

$$\text{at} \quad x = x_1 , \quad \text{(region 1)} \quad \frac{dM_x}{dx} = 0 , \qquad (5.6)$$

$$\text{at} \quad x = x_1 , \quad \text{(region 1)} \quad M_x = - M'_{02} , \qquad (5.7)$$

and
$$\text{at} \quad x = x_2 , \quad \text{(region 1)} \quad M_x = M_{03} \qquad (5.8)$$

Case II

In some cases, the third hinge circle may form in the middle region whereas the second one remains in the bottom region itself. For this case expressions for shear and moment are required for both bottom and middle regions. The simplified equations for shear force and bending moment in the two regions are the following:

region 1:
$$\frac{dM_x}{dx} = \frac{p_o}{2l}(2lx - x^2 - 2lx_2 + x_2^2)$$

$$- \frac{N_{t1}}{R}(x - l_1) - \frac{N_{t2}}{R}(l_1 - x_2) , \qquad (5.9)$$

$$M_x = \frac{p_o}{6l}(3lx^2 - x^3 - 6lx_2x + 3x_2^2x)$$

$$- \frac{N_{t1}}{2R}(x^2 - 2l_1x) - \frac{N_{t2}}{R}(l_1x - x_2x) + M_{01} \,, \tag{5.10}$$

region 2: $$\frac{dM_x}{dx} = \frac{p_o}{2l}(2lx - x^2 - 2lx_2 + x_2^2) - \frac{N_{t2}}{R}(x - x_2) \,, \tag{5.11}$$

and $$M_x = \frac{p_o}{6l}(3lx^2 - x^3 - 6lx_2x + 3x_2^2x)$$

$$- \frac{N_{t2}}{2R}(x^2 - 2x_2x + l_1^2) + M_{01} + \frac{N_{t1}l_1^2}{2R} \tag{5.12}$$

In order to solve for the three unknowns p_o, x_1 and x_2, the following conditions are used:

at $x = x_1$, (region 1) $\frac{dM_x}{dx} = 0$, (5.13)

at $x = x_1$, (region 1) $M_x = -M_{02}'$, (5.14)

and at $x = x_2$, (region 2) $M_x = M_{03}$ (5.15)

Case III

When both second and third hinge circles form in the middle region ($N_\theta = -N_{t2}$), the equations for shear force and bending moment for the two regions will remain exactly the same as those given for Case II. Using these equations, the three unknowns p_o, x_1 and x_2 can be obtained using the foll-owing conditions:

at $x = x_1$, (region 2) $\frac{dM_x}{dx} = 0$, (5.16)

$$\text{at} \quad x = x_1 , \quad \text{(region 2)} \quad M_x = -M'_{02} , \tag{5.17}$$

$$\text{and} \quad \text{at} \quad x = x_2 , \quad \text{(region 2)} \quad M_x = M_{03} \tag{5.18}$$

Application of such conditions, in each of the foregoing cases, will result in three nonlinear simultaneous equations which have been solved numerically using the Newton-Raphson method.

5.5 Optimization

5.5.1 Design variables

All the design variables considered for the Type II towers will be valid for this case also. In addition to these five more design variables, first one for the thickness, t_p, of the partition wall, next three for the percentages of hoop reinforcement, p_{rp1}, p_{rp2} and p_{rp3} and the last one for the percentage of reinforcement, p_{rp4}, in the vertical direction in the cylindrical partition for a short length equal to 1 m near junction 2. Thus, the total number of design variables, to be considered in the optimal design of Type III water towers, is seventeen.

5.5.2 Objective function

The objective function for the case under consideration will include the cost of materials and formwork required for the partition in addition to the costs considered for finding the objective function of the Type II towers. Hence the expression for the objective function in this case will be of the following form:

$$F = F'' + 2\pi R_2 l \; [t_p C_c + \frac{0.25}{100} t_p \; \rho_s \; C_s + 2C_f$$

$$+ \frac{t_p \; \rho_s \; C_s}{300} (p_{rp1} + p_{rp2} + p_{rp3})]$$

$$+ 2\pi R_2 \; (\frac{p_{rp4} - 0.25}{100} t_p \; \rho_s \; C_s) \qquad (5.19)$$

where F" is the value of objective function for the Type II towers as given by Eq. (4.3), and l is the length of the cylindrical partition.

5.5.3 Constraints

All the thirty-five constraints which are considered for the Type II towers will be valid for this case as well. In addition to these, the constraints governing the design of the cylindrical partition are to be incorporated in the present case. The serviceability limit state of cracking demands that the direct and flexural tensile stresses in concrete as well as in steel should remain within prescribed values as stated in Section 3.6.4. Thus, if $N_{\theta p1}$, $N_{\theta p2}$ and $N_{\theta p3}$ represent the maximum hoop tension in regions 1, 2 and 3 respectively and M_{pc} and M_{ps} are the maximum bending moment carrying capacities of unit width section of the partition as per the limiting stresses in concrete and steel respectively, then the constraints thirty-six through forty-three can be written as:

$$N_{\theta p1}/[t_p (\frac{p_{rp1}}{100}) \; f_{st\,max}] - 1 \leq 0 \; , \qquad (5.20)$$

$$N_{\theta p2}/[t_p (\frac{p_{rp2}}{100}) \; f_{st\,max}] - 1 \leq 0 \; , \qquad (5.21)$$

$$N_{\theta p3}/[t_p(\frac{p_{rp3}}{100})\ f_{st\ max}] - 1 \leq 0\ , \tag{5.22}$$

$$\frac{M_{pa}}{M_{pc}} - 1 \leq 0\ , \tag{5.23}$$

$$\frac{M_{pa}}{M_{ps}} - 1 \leq 0\ , \tag{5.24}$$

$$N_{\theta p1}/(A_{eu1}\ f_{ct\ max}) - 1 \leq 0\ , \tag{5.25}$$

$$N_{\theta p2}/(A_{eu2}\ f_{ct\ max}) - 1 \leq 0\ , \tag{5.26}$$

and $$N_{\theta p3}/(A_{eu3}\ f_{ct\ max}) - 1 \leq 0 \tag{5.27}$$

where M_{pa} = actual maximum bending moment per unit width of partition wall in the edge region of junction 2, and

$f_{st\ max}$, $f_{ct\ max}$, and A_{eui} are as defined in the Section 3.6.4 except in finding A_{eui}, t_c and p_{rci} are to be replaced by t_p and p_{rpi} for the present case.

The constraints regarding the minimum thickness and reinforcement for the partition wall can be expressed in the following form:

$$\frac{t_{min}}{t_p} - 1 \leq 0\ , \tag{5.28}$$

$$\frac{p_{r\ min}}{p_{rp1}} - 1 \leq 0\ , \tag{5.29}$$

$$\frac{p_{r\ min}}{p_{rp2}} - 1 \leq 0\ , \tag{5.30}$$

$$\frac{p_{r\ min}}{p_{rp3}} - 1 \leq 0\ , \tag{5.31}$$

and

$$\frac{p_{r\ min}}{p_{rp4}} - 1 \leq 0 \tag{5.32}$$

Thus, for this case, the total number of constraints is forty-eight.

5.6 Parametric Study

Similar to Type II tower, in this case also, optimal limit state design has been carried out for the following cases:

(1) capacity in m^3: 500, 750 and 1000;

(2) staging height in m: 15, 20 and 25; and

(3) basic wind pressure in kN/m^2: 1.0, 1.5 and 2.0.

Suitability of these designs are also established for various seismic zones of India and a free board of about 200 mm has been provided in all the cases.

5.7 Results and Discussions

Similar to the previous two cases, for this type of towers also it is found that for optimal configuration the bending moment and shear force reduce considerably in all the elements in the edge region of junction 2 and the value of maximum hoop tension in conical tank becomes minimum, in most of the cases.

The optimal designs of this type of towers can be obtained from Tables 5.1 through 5.3, for various capacities, staging heights and wind and seismic zones. Whereas, optimal

Table 5.1 Optimal designs of Type III towers with 500 m^3 capacity tank

L	Wind zone	Optimal design variables								Optimal configuration parameters		Optimal cost	Applica to sei zones
	p_b	t_c t_s	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	R_2 $\bar{L}$	R_1/R_2 α		
(m)	(kN/m^2)	(mm)					(mm)				(m)	(rad)	(Rs)	
25	2.0	172 161	1.422 0.334	1.008 1.197	0.690 1.556	0.385 0.750	150 125	0.688 1.250	0.920 0.520	1.785	2.0 9.53	3.5 1.018	278590	I-V
	1.5	171 160	1.432 0.424	1.068 1.132	0.662 1.848	0.300 0.820	175 125	0.550 1.100	0.830 0.450	1.595	1.75 9.91	4.0 1.013	263412	I-V
	1.0	170 174	1.421 0.360	1.109 0.854	0.779 1.689	0.25 0.925	175 125	0.620 0.900	0.670 0.370	1.035	1.50 9.74	5.0 0.908	248756	I-V
20	2.0	170 193	1.456 0.432	1.109 1.211	0.783 1.893	0.25 0.925	175 125	0.620 0.870	0.650 0.350	1.035	1.50 9.74	5.0 0.908	240523	I-V
	1.5	170 185	1.419 0.369	1.101 0.709	0.791 1.552	0.25 0.925	175 125	0.620 0.880	0.670 0.380	1.035	1.50 9.74	5.0 0.908	233456	I-IV
	1.0	171 172	1.437 0.284	1.113 0.528	0.773 1.000	0.25 0.925	175 125	0.620 0.900	0.650 0.370	1.035	1.50 9.74	5.0 0.908	226700	I-IV
15	2.0	170 180	1.439 0.268	1.132 0.562	0.741 1.143	0.25 0.925	175 125	0.620 0.900	0.650 0.350	1.035	1.50 9.74	5.0 0.908	212605	I-V
	1.5	169 182	1.444 0.250	1.125 0.393	0.746 0.824	0.25 0.925	175 125	0.620 0.889	0.644 0.349	1.035	1.50 9.74	5.0 0.908	210459	I-IV
	1.0	169 177	1.435 0.278	1.111 0.318	0.753 0.448	0.25 0.925	175 125	0.620 0.900	0.650 0.350	1.035	1.50 9.74	5.0 0.908	208608	I-III

Table 5.2 Optimal design of Type III towers with 750 m^3 capacity tank

L	Wind zone	Optimal design variables									Optimal configur-ation parameters		Optimal cost	Applical to seis zones
	p_b	t_c t_s	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	R_2 $\bar{L}$	R_1/R_2 α		
(m)	(kN/m^2)	(mm)					(mm)				(m)	(rad)	(Rs)	
25	2.0	226 208	1.433 0.324	1.027 0.834	0.683 1.812	0.25 0.92	175 130	0.55 1.420	1.050 0.580	2.12	2.0 11.30	4.0 1.011	375937	I-V
	1.5	232 229	1.384 0.304	1.102 0.888	0.751 1.397	0.30 0.88	175 130	0.58 1.050	0.825 0.450	1.38	1.75 10.92	5.0 0.875	363059	I-V
	1.0	222 222	1.448 0.271	1.103 0.568	0.710 1.155	0.25 1.03	175 130	0.65 1.330	1.101 0.565	1.97	1.75 11.87	4.5 1.029	350592	I-IV
20	2.0	225 226	1.393 0.293	1.093 0.811	0.692 1.614	0.25 1.03	175 130	0.65 1.396	0.960 0.550	1.97	1.75 11.87	4.5 1.029	336322	I-V
	1.5	236 253	1.328 0.335	1.081 0.717	0.697 1.588	0.25 1.27	200 130	0.58 1.150	0.800 0.400	1.42	1.50 11.76	5.5 0.960	330275	I-IV
	1.0	238 248	1.331 0.288	1.058 0.470	0.691 1.024	0.25 1.27	200 130	0.58 1.05	0.819 0.402	1.42	1.50 11.76	5.5 0.960	322950	I-III
15	2.0	227 233	1.410 0.267	1.117 0.299	0.755 0.671	0.30 0.88	175 130	0.58 1.04	0.900 0.430	1.38	1.75 10.92	5.0 0.875	308754	I-IV
	1.5	225 253	1.424 0.316	1.105 0.509	0.700 0.995	0.25 1.46	200 130	0.65 1.355	0.950 0.520	1.92	1.5 12.97	5.0 1.090	304042	I-IV
	1.0	224 252	1.443 0.253	1.106 0.279	0.702 0.625	0.25 1.46	200 130	0.65 1.400	1.010 0.550	1.92	1.5 12.97	5.0 1.090	300038	I-III

Table 5.3 Optimal design of Type III towers with 1000 m^3 capacity tank

L	Wind zone p_b	Optimal design variables									Optimal configuration parameters		Optimal cost	Applica to seis zones
		t_c t_s	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	R_2 $\bar{L}$	R_1/R_2 α		
(m)	(kN/m^2)	(mm)					(mm)				(m)	(rad)	(Rs)	
25	2.0	268 264	1.398 0.295	1.077 0.769	0.717 1.513	0.25 0.98	175 134	0.60 1.520	1.102 0.607	2.15	2.0 12.23	4.5 0.961	462523	I-V
	1.5	265 280	1.430 0.315	1.097 0.870	0.717 1.676	0.25 1.10	200 130	0.50 1.342	1.00 0.537	2.03	1.75 12.92	5.0 0.998	448462	I-V
	1.0	268 289	1.389 0.278	1.075 0.466	0.727 0.943	0.25 1.10	200 130	0.50 1.350	1.075 0.557	2.03	1.75 12.92	5.0 0.998	438224	I-IV
20	2.0	267 294	1.413 0.324	1.105 0.694	0.708 1.294	0.25 1.10	200 130	0.50 1.376	1.02 0.555	2.03	1.75 12.92	5.0 0.998	421246	I-V
	1.5	267 287	1.438 0.271	1.104 0.480	0.726 1.112	0.25 1.10	200 130	0.50 1.380	1.080 0.577	2.03	1.75 12.92	5.0 0.998	415835	I-IV
	1.0	267 287	1.445 0.269	1.134 0.316	0.727 0.536	0.25 1.10	200 130	0.50 1.360	1.02 0.568	2.03	1.75 12.92	5.0 0.998	408879	I-III
15	2.0	268 305	1.402 0.305	1.091 0.415	0.699 0.582	0.25 1.10	200 130	0.50 1.350	1.02 0.560	2.03	1.75 12.92	5.0 0.998	389264	I-IV
	1.5	268 297	1.394 0.265	1.083 0.297	0.703 0.435	0.25 1.10	200 130	0.50 1.350	1.02 0.560	2.03	1.75 12.92	5.0 0.998	385002	I-III
	1.0	269 286	1.392 0.262	1.087 0.274	0.695 0.317	0.25 1.10	200 130	0.50 1.360	1.02 0.560	2.03	1.75 12.92	5.0 0.998	383030	I-II

Table 5.4 Optimal designs of Type III towers with 500 m^3 capacity tank for non-optimal shaft ra

L (m)	Wind zone p_b (kN/m^2)	R_2 (m)	t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p (mm)	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	R_1/R_2 α (rad)	$\bar{L}$	Optimal cost (Rs)	Applic to sei zones
			Optimal design variables									Optimal configuration parameters			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
25	2.0	1.75	177 192	1.446 0.449	1.104 1.071	0.723 1.775	0.30 0.680	150 125	0.687 0.900	0.641 0.377	1.14	4.5 0.837	9.15	279694	I-V
	1.5	1.5	175 212	1.393 0.460	1.097 1.597	0.730 1.806	0.25 0.925	175 125	0.620 0.880	0.670 0.400	1.035	5.0 0.908	9.74	270469	I-V
		2.0	173 157	1.421 0.292	1.008 0.618	0.690 1.314	0.385 0.750	150 125	0.688 1.250	0.920 0.520	1.785	3.5 1.018	9.53	271154	I-V
	1.0	1.75	171 159	1.432 0.276	1.068 0.592	0.662 1.322	0.30 0.820	175 125	0.55 1.100	0.830 0.450	1.595	4.0 1.013	9.91	257812	I-IV
		2.0	180 153	1.393 0.265	1.094 0.349	0.750 0.637	0.360 0.620	150 125	0.57 1.050	0.722 0.400	1.24	4.0 0.817	8.77	267511	I-IV
20	2.0	1.75	171 164	1.409 0.365	1.110 0.765	0.714 1.587	0.30 0.820	175 125	0.55 1.50	0.830 0.460	1.595	4.0 1.013	9.91	243479	I-V
		2.0	180 154	1.393 0.297	1.094 0.441	0.750 0.924	0.360 0.620	150 125	0.57 1.050	0.722 0.400	1.24	4.0 0.817	8.77	253605	I-IV
	1.5	1.75	170 158	1.409 0.333	1.057 0.637	0.682 1.121	0.30 0.820	175 125	0.55 1.135	0.802 0.474	1.595	4.0 1.013	9.91	236362	I-IV

Contd...

Table 5.4 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		2.0	179 152	1.407 0.270	1.144 0.397	0.760 0.576	0.360 0.620	150 125	0.57 1.050	0.722 0.400	1.24	4.0 0.817	8.77	250864	I-IV
	1.0	1.75	172 157	1.405 0.278	1.050 0.351	0.680 0.700	0.30 0.820	175 125	0.55 1.150	0.830 0.460	1.595	4.0 1.013	9.91	233329	I-IV
		2.0	179 150	1.407 0.268	1.104 0.297	0.745 0.384	0.360 0.620	150 125	0.57 1.050	0.722 0.400	1.24	4.0 0.817	8.77	245697	I-III
15	2.0	1.75	171 163	1.402 0.303	1.060 0.427	0.704 0.798	0.30 0.820	175 125	0.55 1.150	0.830 0.460	1.595	4.0 1.013	9.91	218202	I-V
		2.0	179 152	1.442 0.267	1.124 0.301	0.745 0.369	0.360 0.620	150 125	0.57 1.050	0.722 0.400	1.24	4.0 0.817	8.77	228936	I-IV
	1.5	1.75	172 158	1.402 0.279	1.050 0.360	0.682 0.556	0.30 0.820	175 125	0.55 1.135	0.820 0.460	1.595	4.0 1.013	9.91	217006	I-IV
		2.0	179 150	1.442 0.264	1.120 0.284	0.735 0.307	0.360 0.620	150 125	0.57 1.050	0.722 0.405	1.24	4.0 0.817	8.77	228338	I-IV
	1.0	1.75	171 157	1.410 0.251	1.064 0.272	0.675 0.306	0.30 0.820	175 125	0.55 1.135	0.830 0.460	1.595	4.0 1.013	9.91	213550	I-III
		2.0	179 151	1.443 0.268	1.081 0.279	0.730 0.290	0.360 0.620	150 125	0.57 1.050	0.722 0.400	1.24	4.0 0.817	8.77	228947	I-III

Table 5.5 Optimal designs of Type III towers with 750 m^3 capacity tank for non-optimal shaft rad.

L	Wind zone p_b	R_2	Optimal design variables								Optimal configuration parameters		Optimal cost	Applical to seisı zones
			t_c t_s	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	R_1/R_2 α	$\bar{L}$		
(m)	(kN/m^2)	(m)	(mm)					(mm)				(rad)		(Rs)	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
25	2.0	2.25	237 200	1.424 0.258	1.091 0.411	0.707 1.086	0.25 0.77	150 130	0.68 1.154	0.942 0.470	1.77	4.0 0.840	10.11	384210	I-V
		1.75	227 241	1.441 0.435	1.129 1.281	0.771 1.771	0.30 0.88	175 130	0.58 1.040	0.800 0.430	1.38	5.0 0.875	10.92	377610	I-V
	1.5	1.5	238 305	1.320 0.439	1.044 1.148	0.688 1.865	0.25 1.27	200 130	0.58 1.050	0.819 0.402	1.42	5.5 0.960	11.76	375155	I-V
		2.0	226 209	1.420 0.269	1.085 0.578	0.683 1.224	0.25 0.92	175 130	0.55 1.380	1.011 0.56	2.12	4.0 1.011	11.30	365185	I-IV
	1.0	1.5	241 244	1.304 0.268	1.026 0.875	0.680 1.895	0.25 1.27	200 130	0.58 1.070	0.780 0.440	1.42	5.5 0.960	11.76	351514	I-IV
		2.0	224 209	1.443 0.290	1.080 0.356	0.699 0.684	0.25 0.92	175 130	0.55 1.380	1.011 0.56	2.12	4.0 1.011	11.30	358624	I-IV
20	2.0	1.5	236 270	1.360 0.412	1.080 1.439	0.688 1.667	0.25 1.27	200 130	0.58 1.070	0.780 0.440	1.42	5.5 0.960	11.76	342358	I-V
		2.0	226 216	1.398 0.320	1.085 0.517	0.700 1.002	0.25 0.92	175 130	0.55 1.380	1.011 0.56	2.12	4.0 1.011	11.30	343284	I-V

Contd...

Table 5.5 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	1.5	1.75	228 230	1.381 0.269	1.140 0.648	0.698 0.961	0.25 1.03	175 130	0.65 1.325	0.954 0.500	1.97	4.5 1.029	11.87	332218	I-IV
		2.0	228 207	1.392 0.282	1.073 0.347	0.700 0.706	0.25 0.92	175 130	0.55 1.380	1.011 0.560	2.12	4.0 1.011	11.30	337061	I-III
	1.0	1.75	227 226	1.428 0.252	1.128 0.318	0.753 0.511	0.30 0.88	175 130	0.58 1.020	0.780 0.420	1.38	5.0 0.875	10.92	326526	I-III
		2.0	226 209	1.404 0.253	1.030 0.270	0.669 0.369	0.25 0.92	175 130	0.55 1.380	1.011 0.560	2.12	4.0 1.011	11.30	331439	I-III
15	2.0	1.5	236 273	1.360 0.360	1.065 0.609	0.717 0.870	0.25 1.27	200 130	0.58 1.100	0.800 0.440	1.42	5.5 0.960	11.76	308936	I-V
		2.0	223 212	1.474 0.290	1.125 0.355	0.680 0.456	0.25 0.92	175 130	0.55 1.380	1.000 0.580	2.12	4.0 1.011	11.30	314250	I-IV
	1.5	1.75	227 229	1.386 0.315	1.107 0.376	0.727 0.488	0.25 1.03	175 130	0.65 1.325	0.954 0.500	1.97	4.5 1.029	11.87	307206	I-IV
		2.0	224 205	1.440 0.265	1.060 0.266	0.680 0.334	0.25 0.92	175 130	0.55 1.380	1.033 0.570	2.12	4.0 1.011	11.30	309436	I-III
	1.0	1.75	223 218	1.425 0.270	1.074 0.311	0.700 0.377	0.25 1.03	175 130	0.65 1.325	0.954 0.500	1.97	4.5 1.029	11.87	302148	I-III
		2.0	225 195	1.407 0.252	1.040 0.259	0.670 0.271	0.25 0.92	175 130	0.55 1.391	1.011 0.540	2.12	4.0 1.011	11.30	306456	I-III

Table 5.6 Optimal designs of Type III towers with 1000 m^3 capacity tank for non-optimal shaft radi

L	Wind zone p_b	R_2	Optimal design variables									Optimal configuration parameters		Optimal cost	Applicabl to seismi zones
			t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p (mm)	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	R_1/R_2 α (rad)	$\bar{L}$	(Rs)	
(m)	(kN/m^2)	(m)													
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
25	2.0	2.25	275 243	1.439 0.275	1.024 0.426	0.695 1.380	0.30 0.98	175 130	0.58 1.600	0.670	2.45	4.0 0.965	11.85	475360	I-V
		1.75	263 321	1.467 0.456	1.134 1.266	0.747 1.715	0.25 1.10	200 130	0.50 1.360	1.020 0.560	2.03	5.0 0.998	12.92	474839	I-V
	1.5	1.5	286 360	1.308 0.686	1.028 1.481	0.654 1.940	0.25 1.62	200 130	0.78 1.389	1.000 0.530	2.08	5.5 1.075	14.19	480900	I-V
		2.0	267 258	1.426 0.285	1.096 0.512	0.727 1.163	0.25 0.98	175 130	0.60 1.406	1.072 0.557	2.15	4.5 0.961	12.23	452250	I-IV
	1.0	1.5	286 320	1.309 0.314	1.017 0.881	0.654 1.703	0.25 1.62	200 130	0.78 1.400	1.020 0.529	2.08	5.5 1.075	14.19	450325	I-IV
		2.0	264 270	1.473 0.278	1.106 0.375	0.729 0.581	0.25 0.98	175 130	0.60 1.440	1.100 0.560	2.15	4.5 0.961	12.23	450058	I-IV
20	2.0	1.5	286 353	1.308 0.508	1.028 1.376	0.654 1.768	0.25 1.62	200 130	0.78 1.390	1.020 0.530	2.08	5.5 1.075	14.19	447203	I-V
		2.0	260 265	1.427 0.345	1.096 0.508	0.725 0.990	0.25 0.98	175 130	0.60 1.440	1.100 0.560	2.15	4.5 0.961	12.23	426907	I-V

Contd...

Table 5.6 (continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	1.5	1.5	286	1.320	1.034	0.655	0.25	200	0.78	1.020	2.08	5.5	14.19	428138	I-IV
			336	0.417	0.855	1.478	1.62	130	1.400	0.530		1.075			
		2.0	267	1.425	1.100	0.725	0.25	175	0.60	1.100	2.15	4.5	12.23	423201	I-IV
			263	0.311	0.359	0.673	0.98	130	1.440	0.560		0.961			
	1.0	1.5	286	1.310	1.047	0.658	0.25	200	0.78	1.020	2.08	5.5	14.19	420845	I-III
			353	0.301	0.488	0.852	1.62	130	1.400	0.530		1.075			
		2.0	269	1.392	1.095	0.715	0.25	175	0.60	1.100	2.15	4.5	12.23	418241	I-III
			263	0.264	0.279	0.316	0.98	130	1.440	0.560		0.961			
15	2.0	1.5	288	1.295	1.017	0.646	0.25	200	0.78	1.034	2.08	5.5	14.19	398103	I-V
			341	0.297	0.653	1.161	1.62	130	1.415	0.530		1.075			
		2.0	266	1.437	1.110	0.737	0.25	175	0.60	1.100	2.15	4.5	12.23	394918	I-IV
			263	0.277	0.326	0.471	0.98	130	1.440	0.560		0.961			
	1.5	1.5	286	1.340	1.017	0.648	0.25	200	0.78	1.020	2.08	5.5	14.19	395770	I-IV
			348	0.328	0.498	0.731	1.62	130	1.410	0.530		1.075			
		2.0	265	1.470	1.114	0.738	0.25	175	0.60	1.100	2.15	4.5	12.23	393638	I-III
			259	0.265	0.274	0.370	0.98	130	1.440	0.560		0.961			
	1.0	1.5	287	1.303	1.010	0.640	0.25	200	0.78	1.020	2.08	5.5	14.19	387901	I-III
			328	0.276	0.369	0.491	1.62	130	1.410	0.530		1.075			
		2.0	271	1.383	1.071	0.719	0.25	175	0.60	1.100	2.15	4.5	12.23	390927	I-II
			244	0.258	0.306	0.364	0.98	130	1.440	0.560		0.961			

Table 5.7 Effect of cost ratios on optimal configuration

Capacity of tank = 1000 m^3, L = 25 m, p_b = 2.0 kN/m^2

S. No.	CR1	CR2	R_2 (m)	R_1/R_2	Optimal cost (Rs)	Optimal configuration
1	1.25	0.10	1.75	4.5	500601	
				5.0	474839	
				5.5	477288	R_2 = 2.0 m,
			2.0	4.0	486777	
				4.5	462523	R_1/R_2 = 4.5 or
				5.0	476437	
			2.25	3.5	502807	α = 0.961
				4.0	475360	
				4.5	486921	
2	2.0	0.10	1.75	4.5	622368	
				5.0	590239	
				5.5	598344	R_2 = 2.0 m,
			2.0	4.0	602469	
				4.5	577346	R_1/R_2 = 4.5 or
				5.0	598896	
			2.25	3.5	621464	α = 0.961
				4.0	589094	
				4.5	615327	
3	1.25	0.15	1.75	4.5	565562	
				5.0	542790	
				5.5	540032	R_2 = 2.0 m,
			2.0	4.0	556466	
				4.5	530409	R_1/R_2 = 4.5 or
				5.0	546127	
			2.25	3.5	566368	α = 0.961
				4.0	544627	
				4.5	560431	

Table 5.8 Effect of cost ratios on optimal design variables

Capacity of tank = 1000 m^3, L = 25 m, p_b = 2.0 kN/m^2

S. No.	R_2 (m)	R_1/R_2	CR1	CR2	Optimal design variables									Optimal cost (Rs)
					t_c t_s (mm)	p_{rc1} p_{rs1}	p_{rc2} p_{rs2}	p_{rc3} p_{rs3}	p_{rc4} p_{rcy}	t_{bd} t_p (mm)	p_{rbd} p_{rp1}	p_{rp2} p_{rp3}	p_{rp4}	
1	2.0	4.5	1.25	0.10	268 264	1.398 0.295	1.077 0.769	0.717 1.513	0.25 0.98	175 134	0.60 1.520	1.102 0.607	2.15	462523
2	2.0	4.5	2.00	0.10	264 252	1.490 0.268	1.098 0.899	0.751 1.710	0.25 0.98	175 130	0.60 1.538	1.141 0.610	2.15	577346
3	2.0	4.5	1.25	0.15	264 261	1.494 0.304	1.090 0.820	0.753 1.629	0.25 0.98	175 130	0.60 1.540	1.106 0.620	2.15	530409
4	2.0	4.5	0.75	0.10	265 254	1.453 0.282	1.112 0.890	0.732 1.665	0.25 0.98	175 130	0.60 1.540	1.112 0.620	2.15	390566

Table 5.9 Capacity of inner portion of the tank for different cases

Tank capacity (m^3)	R_2 (m)	α (radian)	Height of partition wall l (m)	Capacity of the inner portion (m^3)
500	1.50	0.908	7.679	40.71
	1.75	0.837	6.792	53.34
		1.013	8.410	66.05
	2.00	0.817	6.397	69.08
		1.018	8.109	87.57
750	1.50	0.960	9.641	51.11
		1.090	11.510	61.03
	1.75	0.875	8.388	65.88
		1.029	10.173	79.90
	2.00	1.011	9.571	103.36
	2.25	0.840	7.530	106.45
1000	1.50	1.075	12.490	66.21
	1.75	0.998	10.864	85.32
	2.00	0.961	10.020	108.21
	2.25	0.965	9.740	137.70

designs for these cases for non-optimal radii of the supporting shaft are given in Tables 5.4 through 5.6. It can be observed from the results that the optimal value of R_2 and α for a particular capacity, staging height, and wind and seismic zones, for this case, is found to be less as compared to that of Type II towers. This is due to the fact that the cost of cylindrical partition reduces with decrease in the value of R_2 and α both.

For this type of towers the optimal value of α lies between 0.817 to 1.090 radian in all the cases but for most of these it is between 0.9 to 1.03 radian. The optimal value of R_2 for most of the cases of 500 and 750 m^3 capacity tanks is 1.5 m, whereas it is 1.75 m for the towers with 1000 m^3 capacity tank.

Discussion on relative values of wind and seismic forces for various cases, effect of cost ratios on optimal configuration and the design variables as well as those dealing with constraints which are active, given in Sections 3.9 and 4.6, is found to be valid for this type of towers also. In addition to these, the following constraints, all pertaining to partition wall, are also critically satisfied in the optimal design of this type of towers:

(1) the constraint dealing with minimum thickness,

(2) all the constraints corresponding to maximum allowable stress in hoop reinforcement,

(3) the constraint which deals with maximum allowable direct tensile stress in concrete in the region of maximum hoop tension, and

(4) the constraint in relation to maximum allowable flexural tensile stress in steel in the boundary region of junction 2.

The capacity of the inner portion of the tank depends upon the value of R_2 and α. The Table 5.9 gives this for different capacity, R_2 and α values. It can be seen that the capacity of inner portion for 500, 750 and 1000 m^3 capacity tanks lies between 40.71 to 87.57, 51.11 to 106.45 and 66.21 to 137.70 m^3 respectively for different values of R_2 and α in each case. For a design case in which there is no specific requirement regarding the capacity of inner portion, the relevant optimal designs, from one of the Tables 5.1 through 5.3, can be picked up. When the design of a water tower with a specific capacity of the inner portion, which lies between the range given above, is to be carried out, then a suitable design with optimal value of R_2 or otherwise can be obtained from Tables 5.1 through 5.6. However, these designs are not applicable to the cases in which the required capacity of the inner portion does not lie between the corresponding range prescribed in the foregoing discussion.

5.8 Variable Thickness of the Conical Tank Wall

Similar to the case of Type II towers thickness of the conical tank wall can be varied without any error in the governing design forces and without violating the design constraints. The discussion and procedure regarding variation of

thickness of the tank wall, described in Section 4.7, will also be valid for this type of towers.

5.9 Example

Similar to the other two cases, to illustrate the use of design tables, an example has been considered for this case also.

Problem: A conical water tank supported on cylindrical shaft is required to be designed for a capacity of 1000 m^3, staging height 25 m and wind and seismic zones I and II respectively, when the capacity of the inner portion of the tank is required to be not less than (a) 75 m^3 and (b) 100 m^3.

(a) It can be seen from Table 5.9 that for 1000 m^3 capacity tank, the required capacity of 75 m^3, of the inner portion is available for three values of R_2 which are 1.75, 2.0 and 2.25 m. However, out of these three, R_2 = 1.75 m corresponds to the optimal radius of the supporting shaft for the given capacity, staging height and the wind and seismic zones (Table 5.3). Hence the optimal design values obtained from the Table 5.3 are as follows:

Radius of the supporting shaft 'R_2' = 1.75 m.

Value of R_1/R_2 = 5.0 (α = 0.998).

Length of the conical tank wall '$\bar{L}$' = 12.92 m.

Thickness of the conical tank wall 't_c' = 268 mm.

Areas of hoop reinforcement in the three regions of the conical portion (Figure 3.15) in order are 3722, 2881 and 1953 mm^2/m.

Area of meridional reinforcement in the conical portion (as per minimum requirement of 0.25%) = 670 mm^2/m.

Thickness of the supporting shaft 't_s' = 289 mm.

Areas of vertical reinforcement in the three regions of the supporting shaft (Figure 3.16) in order are 803, 1346 and 2725 mm^2/m.

Area of circumferential reinforcement in the supporting shaft (as per the minimum requirement of 0.25%) = 722.5 mm^2/m.

Thickness of the bottom dome = 200 mm.

Area of reinforcement in meriodional as well as in circumferential direction, for the bottom dome, is = 1000 mm^2/m.

Thickness of the cylindrical partition wall 't_p' = 130 mm.

Areas of hoop reinforcement in the three regions of the partition wall (Section 5.3) in order are 1755, 1397 and 698 mm^2/m.

Area of vertical reinforcement in the partition wall for a short length of 1 m (as per percentage of reinforcement 'p_{rp4}') = 2639 mm^2/m.

Area of vertical reinforcement in the partition wall for the remaining length (as per minimum requirement of 0.25%) = 325 mm^2/m.

A 100 mm thick top dome with minimum reinforcement of 250 mm^2/m in both the directions will be adequate for all the cases. The inner cylindrical shaft of 125 mm thickness with minimum reinforcement = 312.5 mm^2/m, in the two directions

is found to be adequate for all the cases except for a very short length of 1 m near junction 2 in which the vertical reinforcement (as per the percentage of reinforcement given by 'p_{rcy}') for each case will be different. For this case it is = 1375 mm^2/m.

Ring beams of 400 x 400 mm and 225 x 225 mm sizes with 0.5% hoop reinforcement will be adequate enough for junctions 1 and 2, and 4 respectively.

The thickness of the conical tank wall can be varied as per the procedure described in Section 4.7. The thickness from junction 2 to 0.5 $\bar{L}$ (6.46 m) will remain equal to 268 mm itself. From this point to the point of junction 1, the thickness can be decreased uniformly to a minimum of 125-150 mm. The area of hoop reinforcement in each of three regions of the conical portion will essentially remain unchanged and will be equal to 3722, 2881 and 1953 mm^2/m respectively. However, a slightly thicker section with gradually increasing thickness for a short length near junction 1 should be provided.

(b) In this case the capacity of inner portion of the tank is required to be at least 100 m^3, for which possible radii of the supporting shaft are 2.0 and 2.25 m (Table 5.9). However, the optimal design with R_2 = 2.25 m is not given for this particular case as it will be very much uneconomical. Hence the only value of R_2 which will provide the required capacity of inner portion is = 2.0 m. This value of R_2 does not correspond to the optimal radius for the case

under consideration. Therefore, the optimal designs for non-optimal supporting shaft radii, given in Table 5.6, will be adopted for this case. Adopting the foregoing procedure, the design details of the tower can be obtained without any difficulty for this case also.

CHAPTER 6

COMPUTATIONAL ASPECTS

6.1 General

The developments of modern science have confronted the scientist and the engineer with a variety of problems which are best solved by approximate methods. Mathematical programming techniques, with the exception of classical methods, involve lot of numerical computations. Numerical analysis has been applied to scientific and technological problems from the very beginning of applied science, but has been given new impetus by the development of electronic digital computer. In this chapter, computational details of the optimization techniques used in this study, organisation of the computer programs, and related aspects are presented. All the computations have been carried out on DEC-1090 system at the Indian Institute of Technology, Kanpur.

6.2 Algorithms for the Optimization Techniques

The formulated constrained nonlinear programming problems of the present study have been transformed into equivalent unconstrained nonlinear programming problems through the use of interior penalty functions as described in Section 1.4.5. In order to find the minimum of the modified objective function, Davidon-Fletcher-Powell method (DFP method) is used to find search direction $\bar{S}$ and cubic interpolation method is employed to find the appropriate

step length α. The various steps involved in the solution of the constrained optimization problem may be summarised as follows:

(1) The solution is commenced from a feasible point $\bar{X}_1$. A suitable value is chosen for the penalty parameter r and the counter k set equal to 1.

(2) Using DFP algorithm and cubic interpolation method as explained in steps (6) through (11), the vector $\bar{X}_k^*$ is obtained which minimizes the modified objective function $\phi\ (\bar{X}, r_k)$.

(3) The point $\bar{X}_k^*$ is tested for optimality. If it turns out to be optimal, the process is terminated.

(4) Otherwise, the next value of penalty parameter is obtained from the relation

$$r_{k+1} = cr_k , \quad \text{where } c \text{ is less than } 1.$$

(5) The new value of k is set equal to k+1, the new starting point $\bar{X}_1$ set equal to $\bar{X}_k^*$ and the next minimization cycle starts from step (2).

The DFP algorithm and the cubic interpolation method can be summarised as follows:

(6) The algorithm starts with the initial point $\bar{X}_1$ and a n x n positive definite symmetric matrix $[H]_1$, where n is the number of design variables. The iteration number i is set equal to 1.

(7) The gradient of the modified objective function, ϕ_1, at the point $\bar{X}_1$ is computed, from which the

search direction $\bar{S}_i$ is found as

$$\bar{S}_i = - [H]_i \nabla \phi_i \tag{6.1}$$

(8) The minimizing step length α_i^* in the direction $\bar{S}_i$ is found using the cubic interpolation method. The improved design point is then found as

$$\bar{X}_{i+1} = \bar{X}_i + \alpha_i^* \bar{S}_i \tag{6.2}$$

(9) The new point $\bar{X}_{i+1}$ is tested for optimality. If it is found to be optimal, the iterative procedure is terminated.

(10) Otherwise, the $[H]_i$ matrix is updated as

$$[H]_{i+1} = [H]_i + [M]_i + [N]_i \tag{6.3}$$

where $$[M]_i = \alpha_i^* \frac{\bar{S}_i \bar{S}_i^T}{\bar{S}_i^T \bar{Q}_i}, \tag{6.4}$$

$$[N]_i = - \frac{([H]_i \bar{Q}_i)([H]_i \bar{Q}_i)^T}{\bar{Q}_i^T [H]_i \bar{Q}_i} \tag{6.5}$$

and $$\bar{Q}_i = \nabla \phi_{i+1} - \nabla \phi_i \tag{6.6}$$

(11) The new iteration number i is set equal to i+1 and the iteration started from step (7).

6.3 Organisation of the Computer Programs

The computer programs used in this study can be broadly classified into four categories; main program, optimization subroutines, function subroutines and the function

subprograms. The main program essentially reads the input data, calls the relevant subroutines, and prints the output. The principal subroutines used to carry out the optimization are MINIM4 and STEPX3, the former to find out the search direction and the latter to obtain the minimizing step length. In order to find out the gradient of the modified objective function and the directional derivative, these subroutines in turn call GRADN and SLOPE2. The subroutine SLOPE2 also calls GRADN. The subroutine GRADN makes use of the finite difference approach to find the gradient vector. Since the gradient, in the finite difference method, is computed on the basis of function value, subroutine GRADN in turn calls FTN. The purpose of the subroutine FTN is to find the value of the objective function, all the constraints g_j $(j = 1,2,...m)$, and the modified objective function ϕ for a chosen value of the penalty parameter. Thus, subroutine FTN in turn calls OBJT and CONST. The constraints can only be evaluated after carrying out the elastic and limit analysis of the tower. Thus, the subroutine CONST in turn calls ELANL and LIMANL. These in turn make use of the following subroutines: MATINV to obtain the inverse of a matrix for solving a set of linear simultaneous equations for finding the design forces in the reservoir portion, RAPSON to solve a set of nonlinear simultaneous equations for limit analysis of the cylindrical partition, RECT1 to find the ultimate strength of a rectangular section, and CIRCU1 to obtain the ultimate strength of circular cross section of the supporting shaft. Function

subprograms are employed by subroutines RECT1 and CIRCU1 to find the value of functions for obtaining the values of neutral axis and moment of inertia.

6.4 Some Important Aspects of Computer Programs

Several precautions are necessary for obtaining the satisfactory results through computer programs, particularly, when optimization technique is incorporated in the design process. Based upon the observations made during this study and the experience of others (Subramanyam, 1981; Srinivasa Rao, 1983), the following precautions are considered to be the important ones:

(1) While carrying out the elastic analysis through force method, described in Section 3.3.1, it has been observed that satisfactory results can only be obtained when double precision is employed during computations. When analysis is carried out adopting single precision it has been found that the values of stress resultants suddenly jump at some points, in some cases. The Table 6.1 shows the results of the analysis through force method for a tower with the following data: Capacity of the tank = 300 m^3, R_2 = 1.125 m, and R_1/R_2 = 4.0. The table shows the values of stress resultants, along the wall of the conical tank, which are obtained using single as well as double precision. Thus, it can be seen that the solution obtained by single precision may lead to a serious error in the results. This happens due to the way in which the

Table 6.1 Results of analysis through force method using single and double precision

$\frac{x}{L}$	Values of stress resultants obtained using single precision (SP) and double precision (DP)					
	N_x in kN/m		N_θ in kN/m		M_x in kN m/m	
	(SP)	(DP)	(SP)	(DP)	(SP)	(DP)
0.00	-614.13	-614.13	163.16	163.16	1.9425	1.9425
0.05	-511.33	-511.33	164.79	164.79	-0.083	-0.083
0.10	-429.93	-429.93	184.49	184.49	-0.025	-0.025
0.15	-363.88	-363.88	193.76	193.76	0.004	0.004
0.20	-309.23	-309.23	201.24	201.24	-0.000	-0.000
0.25	-263.22	-263.22	206.47	206.47	0.000	-0.000
0.30	-223.99	-223.99	209.33	209.33	0.000	0.000
0.35	-190.22	-190.22	209.86	209.86	-0.000	0.000
0.40	-160.93	-160.96	208.04	208.04	0.026	-0.000
0.45	-136.49	-136.49	209.75	203.87	0.000	0.000
0.50	-113.27	-113.27	103.39	197.37	-0.000	0.000
0.55	-105.11	-93.88	940.41	188.52	0.000	0.000
0.60	-77.00	-77.00	3184.8	177.34	0.000	0.000
0.65	-62.35	-62.35	163.81	163.81	439.75	-0.000
0.70	-1955.3	-49.73	147.94	147.94	-0.000	-0.000
0.75	-38.96	-38.96	129.72	129.72	0.000	0.000
0.80	-29.90	-29.90	109.17	109.17	-0.000	-0.000
0.85	-22.44	-22.44	86.25	86.25	-0.000	-0.000
0.90	-16.48	-16.48	Very high	61.06	0.005	0.005
0.95	-11.90	-11.90	34.42	34.42	-0.019	-0.023
1.00	-8.75	-8.75	Very high	-3.89	-0.178	-0.178

arithmetic operations are performed by the computer. Particularly in this case it is likely to happen because some of the elements of the flexibility matrix are of the order of 10^{-8} to 10^{-16}. Hence double precision should be employed while using force method of analysis for shell structures to overcome such difficulties.

(2) All the constraints should be normalized so that their values vary between -1 and 0. If the constraints are not normalized then the contribution of the various constraints may be very much different in the formulation of the Ø-function. This will badly affect the convergence rate during the minimization of Ø-function. In view of the foregoing fact all the constraints in the present study have been expressed in their normalized form.

(3) Too much difference in scale of the variables may cause some difficulties while selecting increments for step lengths or calculating numerical derivatives. Sometimes the objective function contours get distorted due to these scale disparities. Hence it is a good practice to scale the variables so that all the variables become dimensionless and vary between a short range. However, the need of scaling the variables in the present study is not felt as the value of all these variables is expected to lie between 0.125 to 2.0 approximately. It should be noted that the thicknesses of various elements are taken in metres in optimal

design process, whereas, these are given in mm in the design tables for convenience.

(4) The rate of convergence during the minimization of the $\emptyset$-function is found to be very much sensitive to the initial value of penalty parameter r_1 and to some extent to reduction factor c. In the present study, a value of r_1, which gives the value of $\emptyset(\overline{X}_1, r_1)$ approximately equal to 1.2 to 1.75 times the value of $f(\overline{X}_1)$ is found to be quite satisfactory in most of the cases. Whereas, a value of 0.1 has been proved to be satisfactory for the reduction factor c in this study.

(5) All the techniques available for the solution of non-linear programming problems guarantee, at least, a local minimum. This would also be the global minimum, if the objective function is convex. It is not easy to ascertain whether the objective function is convex or not, for the nature of functions encountered in this study or in other real life problems. To get over such a situation, the optimum solution is obtained with different starting points and the solutions so obtained are compared. If the solutions match then it is very likely, but not certain, that the solutions correspond to the global minimum. The Table 6.2 shows the optimal designs of a water tower, obtained from three different starting points. It can be seen that there is no significant difference in the solutions obtained. In the present study, as all the problems solved are

Table 6.2 Optimal design of a water tower from different starting points

Capacity = 500 m^3, R_2 = 1.75 m, R_1/R_2 = 4.0, p_b = 1.5 kN/m^2, L = 20 m

Nomenclature	t_c (m)	p_{rc1}	p_{rc2}	p_{rc3}	p_{rc4}	t_s (m)	p_{rs1}	p_{rs2}	p_{rs3}	p_{rsy}	t_{bd} (m)	p_{rbd}	Optimal cost (Rs)
First starting point	0.250	1.400	1.200	0.900	0.600	0.360	0.900	1.500	1.800	0.500	0.250	0.750	-
Optimal design 1	0.168	1.454	1.074	0.679	0.250	0.156	0.284	0.541	1.156	0.250	0.150	0.500	205791
Second starting point	0.450	1.350	0.750	0.500	0.750	0.500	0.800	1.500	1.750	0.600	0.350	0.900	-
Optimal design 2	0.172	1.377	1.015	0.660	0.250	0.153	0.272	0.566	1.185	0.250	0.150	0.500	205736
Third starting point	0.400	1.250	0.900	0.700	0.500	0.400	1.000	1.300	1.850	0.700	0.270	0.750	-
Optimal design 3	0.171	1.434	1.020	0.680	0.250	0.150	0.280	0.596	1.215	0.250	0.150	0.500	205527

of the same nature, the possibility of having missed the global minimum is practically nil.

(6) In almost all constrained optimization problems, there will be some constraints which are critically satisfied. Since the penalty term tends to infinity as the solution approaches the constraint boundaries, special precautions have to be taken to overcome the overflow problem.

(7) In order to avoid unnecessary large number of unconstrained minimizations as well as the premature termination, several convergence criteria have been employed in the optimization process. These include comparison of the relative values of the objective functions, the design variables, the gradient and related quantities, of the two successive iterations. In order to make use of the solutions so obtained with confidence, and to test for optimality, perturbation of the design vector and testing the Kuhn-Tucker conditions can be resorted to.

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Summary

The main aim of this thesis has been to investigate the optimal limit state design of reinforced concrete, shaft supported, conical water tanks for Indian environmental conditions. The emphasis has been to obtain practically usable optimal cost designs, conforming to relevant codes of practice, which can be useful to a professional engineer.

The analysis and design have been carried out using modern techniques which allow a useful insight into the structural behaviour and design aspects of the structures considered in the present study under the expected loads. Economy and efficiency of these structures are ensured through optimization in which all the strength and serviceability constraints of the relevant limit states are incorporated in the formulated optimization problem. The optimization study of these water towers gives both optimal configuration and the optimal values of the design variables. Furthermore, the influence of the escalation in unit costs of concrete, steel and formwork on the optimal configuration and the design variables of the tower has been studied in the present work.

Parametric studies have been conducted for the most commonly used reservoir capacities and staging heights for each of the three types of the water towers shown in Figure 1.1. The basic wind pressure is also treated as a parameter

in the study of optimal designs of these water towers. The optimal designs so obtained are finally checked to ensure their safety and suitability for various seismic zones of India.

Finally, the designs are presented in the tabular form, for each of the three types of towers, from which the optimal design variables and the parameters fixing the optimal configuration can be picked up for a particular tank capacity, staging height, and wind and seismic zones.

7.2 Conclusions

From the present study a number of conclusions can be drawn. These are listed as follows:

(1) The use of limit state design philosophy and optimization technique has resulted in economical designs which remain safe and serviceable throughout their design life.

(2) For every tank capacity, staging height, and wind and seismic zones there exist optimal values for the radius of the supporting shaft (R_2), and the slope of the wall of the conical reservoir (α) with the horizontal for which the cost of the superstructure of the water tower is minimum. Similarly for water towers with given radius of supporting shaft, there is an optimal value of slope (α) which gives the minimum cost of the superstructure.

(3) It is quite interesting to note that the optimal value of the slope (α) is found to lie within a

very short range of 0.85 to 0.96, 0.991 to 1.02 and 0.9 to 1.03 radians for Type I, Type II and Type III water towers respectively. Thus, it can be concluded that the most probable optimal value of α will lie between 50° to 60° for all the cases.

(4) The optimal values of R_2 and α for a particular tank capacity, staging height, and wind and seismic zones in case of Type III towers are found to be less as compared to those of Type II water towers, in most of the cases. This is due to the fact that the cost of the partition in Type III towers reduces with decrease in the values of both R_2 and α.

(5) The optimal configuration of the reservoir adjusts itself in a manner that the bending moments and shear forces at the junctions reduce considerably bringing the state of stress in the reservoir portion in the close proximity to that of the membrane state. It is also observed that the value of maximum hoop tension in the conical reservoir becomes the minimum for the optimal configuration, in most of the cases.

(6) The location of the maximum hoop tension in the conical reservoir is invariably found to be between 0.3 $\bar{L}$ to 0.5 $\bar{L}$ (Figure 3.5) from the junction 2, for all the cases of three types of towers considered in the present study. The variation of membrane stresses along the wall of the conical reservoir is also found to be similar in all the cases and the

bending and shear forces remain limited to the boundary region of junctions only.

(7) For a specified cost ratio of concrete to steel, it is observed that the optimal configuration and the design variables are not sensitive to the changes in the cost of formwork. Whereas, for various cost ratios of concrete to steel, some of the optimal design variables do vary but slightly, while the optimal configuration remains essentially unaltered. Thus for all practical purposes both optimal configuration and design variables remain unaffected by any change in the cost of materials. Hence the optimal configuration with corresponding optimal values of the design variables obtained by considering the costs of materials prevailing at the time of investigation will remain optimal for all practical purposes when the costs of materials are different.

(8) In Type III towers the provision of partition has resulted in considerable increase of bending moment values, for the various elements, in the boundary region of junction 2. This is observed, mostly, when only outer portion of the tank is full. However, for the normal case, when both the portions are full, these bending moment values are comparable with those obtained for water towers without partition. The governing design values of hoop tension

for the partition are obtained when inner portion of the tank is full.

(9) The design forces in the reservoir portion are governed by axisymmetric loading combination of dead load and hydrostatic pressure loading under full tank condition. Whereas, lateral forces due to wind and seismic effect under full or empty tank condition are found to be critical for the supporting shaft.

(10) For towers with small capacity tank, the wind forces are found to predominate over the seismic forces for all staging heights considered in this study. It is observed that for 100 to 300 kl capacity tank a tower designed for wind zone I is found to be safe for all the seismic zones of India for almost all heights of staging considered herein. It is also identified that the ratio of seismic forces to wind forces increases with the increase in the capacity of the tank for a particular staging height and it decreases with the increase in the staging height for a given capacity of the tank.

(11) The serviceability limit state of cracking is the only one which is found to govern the design of the reservoir portion in all the cases with the exception of some of the cases of 1000 m^3 capacity tank in which the constraint dealing with the ultimate limit state of strength of a section of the conical tank at junction 2 also becomes critical. In case of supporting shaft, for towers with 100 and 200 m^3

capacity tank, only the serviceability limit state of deflection is found to govern the design. Whereas, for towers with larger capacity tank, both deflection as well as the ultimate strength of a local section of unit width of the supporting shaft become critical.

The resistance of these structures to collapse is invariably found to be more than adequate in all the cases.

(12) The extra cost for the provision of the partition for 500, 750 and 1000 kl capacity tanks has come out to be approximately 11 to 15%, 11 to 14% and 10 to 12% respectively of the cost of the tower without partition. The various benefits of partition, therefore, accrue only by incurring the said extra expenditure.

(13) From the comparative cost study of two types of towers viz. Type I and Type II it is observed that for tank capacities upto 300 kl the difference between the costs of the two towers is marginal (1 to 3.5%). However, for capacities 500 kl and above the Type II towers are quite economical. Although the cost of the Type I towers for capacities upto 300 kl is slightly higher than the Type II towers, from constructional point of view the former are simpler. Hence, in the present study, the preference of Type I towers over Type II upto 300 kl capacity is justifiable.

(14) As the present study addresses itself to the superstructure only, the optimal designs for radii of supporting shaft other than the optimal radius are also given in tabular form. These designs are in addition to that corresponding to the optimal radius and may come out to be economical for a particular foundation condition when the cost of the foundation is also taken into consideration.

(15) Although the present study is confined to Indian environmental conditions, the results can be judiciously utilized for other environments as well.

7.3 Suggestions for Future Work

The following are some of the suggestions which may be considered for future work:

(1) Study of optimal designs of overhead tanks of different shapes for different capacities and staging heights will help in choosing the most economical shape.

(2) The extension of present work for optimal design of overhead conical water tanks with a partition providing a definite capacity of the inner portion is straightforward.

(3) Although, for larger capacity tanks, an indirect procedure given in the present study for tapering the thickness of the wall of conical reservoir works quite satisfactorily and results in considerable

saving of volume of concrete, the optimal designs with tapering thickness of the reservoir using an exact method of analysis can be investigated.

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